MATHEMATICAL PROBLEMS CONNECTED WITH THE BENDING AND BUCKLING OF ELASTIC PLATES

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1. Introduction. As part of his Gibbs Lecture of last year Theodore von Kármán [5] considered briefly a number of nonlinear problems concerning thin elastic plates. This address will include discussion of linear as well as of nonlinear problems concerning elastic plates, with special stress on a particular aspect of the nonlinear problems. However, it is not my intention to give a more or less complete survey of the literature on the subject; my purpose is rather to present in some detail a few specific leading ideas and points of view and to call attention to a number of unsolved problems.

The mathematical problems connected with thin elastic plates belong with the many problems from mechanics which have interested eminent mathematicians and from which mathematical ideas of fundamental importance have originated. These problems are of at least equal interest in engineering. In fact, a considerable part of the recent interest in the subject of thin plates can be attributed to the need for numerical solutions of problems which arise in the design of the thin-walled structures used in aircraft. The latter problems are in the main nonlinear boundary value problems of an involved type about which almost nothing of a general nature is known, and, aside from the technical difficulties encountered in attempting to obtain numerical solutions, such problems present questions of a purely mathematical nature which require elucidation. The latter part of this address is largely concerned with the mathematical problems which arise in this connection.

The first attempts to formulate mathematically the problem of flexure of thin elastic plates were probably inspired by the experimental researches of Chladni in 1787 on the modes of vibration of thin elastic plates. During the next twenty years a number of unsuccessful attacks were made upon the problem by James Bernoulli the Younger and others. In 1809 the French Academy offered a prize for a theory of the vibrating plate, which, after some controversy, was awarded to Sophie Germain in 1815. Lagrange, who was a member of the prize committee, corrected the theory of Sophie Germain and derived the partial differential equation as we know it now. In

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