

rational canonical form is then easily obtained. Examples are given to support the contention that this process simplifies the computation of the rational canonical form of  $A$ . (Received December 15, 1941.)

### ANALYSIS

108. G. E. Forsythe and A. C. Schaeffer: *A remark on Toeplitz matrices.*

A doubly infinite matrix  $(a_{mn})$  is said to be regular if for every sequence  $\{x_n\}$  with limit  $x'$  the corresponding sums  $y_m = \sum_n a_{mn} x_n$  are defined for all  $m$  and have the limit  $x'$ . An apparently more general definition of regularity is that the sums defining  $y_m$  exist for all sufficiently large  $m$ , depending on  $\{x_n\}$ , and have the limit  $x'$ . Tamarkin (this Bulletin, vol. 41 (1935), pp. 241–243) has obtained necessary and sufficient conditions for the second type of regularity. This result is obtained by elementary methods and related topics are discussed. (Received January 23, 1942.)

109. H. L. Garabedian: *Hausdorff integral transformations.*

This paper involves a study of the integral transformation  $v(x) = \int_0^x u(y) d\phi(y/x)$ , defining a method of summation  $(H, \phi(x))$ , where  $u(x)$  is bounded and continuous,  $x \geq 0$ , and where  $\phi(x)$  is either a Hausdorff mass function or satisfies the conditions: (i)  $\phi(x)$  is of bounded variation on the interval  $0 \leq x \leq 1$ , (ii)  $\phi(x)$  is continuous on the interval  $(0, 1)$  except possibly at  $x=1$ , (iii)  $\phi(0)=0$ , (iv)  $\phi(1)=1$ . It is proved that the transformation is regular when and only when  $\phi(x)$  is a Hausdorff mass function, and sufficient conditions involving the Silverman-Schmidt integral equations are obtained in order that  $(H, \phi_1(x)) \supset (H, \phi_2(x))$ , in the case that  $\phi_1(x)$  and  $\phi_2(x)$  satisfy the conditions stated above. These results are extensions of those obtained by Silverman (Transactions of this Society, vol. 26 (1924), pp. 101–112). (Received January 10, 1942.)

110. A. M. Gelbart: *Functions of two variables with bounded real parts in domains not equivalent to the bicylinder.*

Let  $f(z_1, z_2)$  be regular in the interior of a finite four-dimensional domain  $\mathfrak{M}^4$ , bounded by certain analytic hypersurfaces, and in general not equivalent to the bicylinder, and let  $f(z_1, z_2)$  have a bounded real part in  $\mathfrak{M}^4$ . These domains were first considered by Bergman, and are termed by him, domains with distinguished boundary surfaces. An upper bound for  $|f(z_1, z_2)|$  is obtained in terms of only  $\max \operatorname{Re} f(z_1, z_2)$  in  $\mathfrak{M}^4$ ,  $f(0, 0)$  and the domain considered. From a formula for  $\partial^{m+nf}(z_1, z_2) / \partial z_1^m \partial z_2^n$  in  $\mathfrak{M}^4$ , previously obtained by the author (Transactions of this Society, vol. 49 (1941), pp. 199–210), an upper bound is also obtained for  $|\partial^{m+nf}(z_1, z_2) / \partial z_1^m \partial z_2^n|$ , again in terms of only  $\max \operatorname{Re} f(z_1, z_2)$  in  $\mathfrak{M}^4$ ,  $f(0, 0)$  and the domain. These results depend upon the establishment of a form of the Schwarz lemma in  $\mathfrak{M}^4$  for two variables. (Received January 29, 1942.)

111. H. J. Greenberg and H. S. Wall: *Hausdorff means included between  $(C, 0)$  and  $(C, 1)$ .*

It is shown that if  $\phi(u)$  is any function of bounded variation on the interval  $0 \leq u \leq +\infty$  such that  $\phi(+\infty) - \phi(0) = 1$ , then the function  $\alpha(z) = \int_0^\infty d\phi(u) / (1+zu)$  is a regular moment function; and that when  $\phi(u)$  is further restricted to be monotone