

NOTE ON THE COEFFICIENTS OF OVERCONVERGENT POWER SERIES

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M. B. Porter gave the first known example of an overconvergent power series, that is to say, of a power series in the complex variable with finite radius of convergence such that a suitable sequence of partial sums converges uniformly in a region containing in its interior both points inside and points outside the circle of convergence. Bourion has recently published¹ a general exposition of the theory of overconvergence to which the reader is referred for further historical and technical details.

Ostrowski established the surprising result that a power series $\sum_{n=0}^{\infty} a_n z^n$ of which the partial sums $s_{m_k} \equiv \sum_{n=0}^{m_k} a_n z^n$ exhibit overconvergence, can be expressed as the sum of a power series $\sum_0^{\infty} a_n' z^n$ with a larger radius of convergence and a power series of the form

$$(1) \quad \sum_0^{\infty} a_n'' z^n, \quad a_n'' = 0, \quad \text{whenever } m_k < n \leq n_k$$

where n_k and λ are suitably chosen, with $m_k < \lambda n_k$, $0 < \lambda < 1$. Here we have $a_n = a_n' + a_n''$, $a_n' \cdot a_n'' = 0$; the partial sums $s_{m_k}''(z) = \sum_{n=0}^{m_k} a_n'' z^n$ of (1) also exhibit overconvergence.

It is the object of the present note to employ methods already known in the literature to make Ostrowski's result slightly more precise, especially to indicate that in series (1) the gaps cannot be uniquely defined with abrupt initial and terminal elements impossible of alteration by Ostrowski's process of writing the series as the sum of a series with a larger radius of convergence and a series with larger gaps which exhibits overconvergence. The moduli of the coefficients a_n'' must taper off gradually before the gap (m_k, n_k) , and must increase gradually after the end of the gap; this remark is to be understood first in the sense that there is an upper limit to the moduli of the coefficients near the ends of a gap, a limit which increases as one moves away from the gap.

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¹ *L'Ultraconvergence dans les Séries de Taylor*, Actualités Scientifiques et Industrielles, no. 472, Paris, 1937.