there is no projection on (m) to (C), it may be shown that at least either there is no projection on (m) to Y, or else there is no projection on (m) to the complementary subspace of Y in (C). (An illustration of the case where there is no projection on (m) to the complementary subspace in (C) is provided by the case of a finite dimensional $Y \subset (C)$.)

In a paper in preparation on the extension of linear transformations, the writer intends to discuss the questions indicated above, and related questions.

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SEQUENCES OF STIELTJES INTEGRALS[1]

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Statement of results. Sequences of Riemann-Stieltjes integrals[2] have as yet been little studied, only the following fundamental results being known.

THEOREM A (Helly [**2**]). *Let $g_n(x)$ $(n=1, 2, \cdots)$ be an infinite sequence of real functions defined in the finite closed interval $I=(a, b)$ which satisfy the following two conditions*:

$$\text{(1)} \qquad \textit{Total variation of } g_n \textit{ in } I \equiv V_I(g_n) \leqq M, \qquad M \textit{ a fixed constant,}$$

$$\text{(2)} \qquad g_n \to g \textit{ on } I, \qquad n \to \infty;$$

then for any function $f(x)$ continuous in I, we have[3]

$$\text{(3)} \qquad \int f dg_n \to \int f dg.$$

THEOREM B (Shohat [**3**]). *Let $\{g_n\}$ be a sequence of functions monotonic and uniformly bounded in I and such that*

$$\text{(4)} \quad g_n \to g \textit{ on } E, \quad E \textit{ a set dense on } I \textit{ and including the end points } a, b \textit{ of } I,$$

where g is a monotonic function (all the functions g_n, g monotonic in the same sense); *then we have* (3) *for any function $f(x)$ for which*

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[1] Presented to the Society, January 1, 1941.

[2] A discussion of such integrals with references is to be found in [**1**]. (Numbers in brackets refer to the bibliography.)

[3] When the limits of integration are omitted, it is to be understood that they are the end points a, b of I.