arithmetical relations concerning the Bernoulli and allied numbers. It depends mainly on the following idea: Let $a$ and $b$ be rational, with $a \equiv b \pmod{p}$, where $p$ is any prime integer. If $a$ and $b$ are independent of $p$ it then follows, since there is an infinity of primes, that $a = b$. This is applied in connection with wha is perhaps the simplest formula in which a single Bernoulli number appears: $S_n^{(} \equiv pb_n \pmod{p^2}$, where $S_n(p) = 1^n + 2^n + \cdots + (p-1)^n$. The paper starts with simple identities involving $(x^p - 1)/(x-1)$, obtains congruences modulo $p$ or $p^2$ by differentiation and integration, and then makes substitutions for the variables which give congruences involving the Bernoulli numbers. Generalizations of the Staudt-Clausen theorem as well as analogues of the same are obtained. Proofs are given of nearly all the results which were stated without proof in two previous papers by the writer (Proceedings of the National Academy of Sciences, vol. 23 (1937), pp. 555–559; vol. 25 (1939), pp. 197–201). This article will appear in the Transactions of this Society. (Received August 12, 1941.)

## ANALYSIS

### 467. Lipman Bers: *A convergence theorem for analytic functions of two variables.*

Let $M$ be a domain of the four-dimensional $z_1$, $z_2$-space ($z_k = x_k + iy_k, k = 1, 2$) bounded by the two analytic hypersurfaces $E[z_1 \epsilon B(z_2) + c(z_2),\ z_2 = e^{i\theta},\ 0 \leqq \theta < 2\pi]$ and $E[z_1 \epsilon c(z_2),\ |z_2| < 1]$ where $B(z_2)$ is a star domain bounded by the curve $c(z_2) = E(z_1 = h(z_2, \lambda),\ 0 \leqq \lambda < 2\pi]$ and $h(z_2, \lambda)$ is an analytic function of $z_2$, $|z_2| \leqq 1$, for every fixed value of $\lambda$, and is subject to certain additional conditions. Consider a sequence of analytic functions $\{f_n(z_1, z_2)\}$ defined in $M$ and satisfying the condition $\int_0^{2\pi}\int_0^{2\pi} |f[r_1 h(r_2 e^{i\theta}, \lambda), r_2 e^{i\theta}]|^p d\theta d\lambda < K$, $0 < r_k < 1$, $k = 1, 2$, $n = 1, 2, \cdots$, where $p$ is a fixed number greater than 1. It is known that $f_n(z_1, z_2)$ possesses finite sectorial limits ($F_n(\theta, \lambda)$ almost everywhere on the intersection $F = E[z_1 = h(e^{i\theta}, \lambda),\ z_2 = e^{i\theta},\ 0 \leqq \theta < 2\pi,\ 0 \leqq \lambda \leqq 2\pi]$ of the two boundary hypersurfaces of $M$ (see Bergman and Marcinkiewicz, Fundamenta Mathematicae, vol. 33 (1939), pp. 75–94, and a paper by the author to appear in the American Journal of Mathematics). If the sequence $\{F_n(\theta, \lambda)\}$ converges in a set of positive two-dimensional measure, the sequence $\{f_n(z_1, z_2)\}$ converges uniformly in every closed subdomain of $M$. This theorem can be generalized, in a modified form, for more general types of domains. (Received September 27, 1941.)

### 468. A. B. Brown: *Independent parameters for sets of functions.*

The results previously announced for the case of one function of $n$ variables and $m$ parameters (under the title *On the number of independent parameters*, abstract 46-11-485) are now extended to the case of a set of $r$ functions of $n$ variables and $m$ parameters. The methods and results are similar to those for the case of one function. An additional theorem is given, on invariance of the number of parameters in a "complete" set under certain transformations of parameters. Only one paper is offered for publication, covering the more general case announced here, but under the original title. (Received September 11, 1941.)

### 469. E. R. Lorch: *The theory of analytic functions in normed abelian vector rings.* Preliminary report.

A complex vector space $\mathfrak{R}$ for which the commutative multiplication of elements is defined is here called a normed abelian vector ring if the norm satisfies