OSCULATING QUADRICS OF RULED SURFACES IN RECIPROCAL RECTILINEAR CONGRUENCES

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1. **Introduction.** Let x be a general point of an analytic non-ruled surface S referred to its asymptotic net in ordinary projective space. By a line l_1 at the point x we mean any line through the point x and not lying in the tangent plane of the surface at the point x. Dually, a line l_2 is any line in the tangent plane of the surface at the point x but not passing through the point x. The lines l_1, l_2 are called reciprocal lines if they are reciprocal polar lines with respect to the quadric of Lie at the point x. In this case, when the point x varies over the surface S, the lines l_1, l_2 generate two rectilinear congruences Γ_1, Γ_2 which are said to be reciprocal with respect to the surface. If, however, the point x moves along the u-curve, the locus of the line l_1 is a ruled surface $R_1^{(u)}$ of the congruence Γ_1. The osculating quadric along a generator l_1 of the ruled surface $R_1^{(u)}$ is the limit of the quadric determined by the line l_1 through the point x and the lines l_1 through two neighboring points P_1, P_2 on the u-curve as each of these points independently approaches the point x along the u-curve. The quadric thus defined will be denoted by $Q_1^{(u)}$. A second quadric $Q_1^{(v)}$ is determined by three consecutive lines l_1 at points of the v-curve through the point x. Moreover, there are two quadrics, denoted by $Q_2^{(u)}$ and $Q_2^{(v)}$, which are associated with two ruled surfaces of the reciprocal congruence Γ_2 and which can be defined similarly. This note will study the projective differential geometry of the quadrics thus defined.

2. **Analytic basis.** Let the surface S under consideration be an analytic non-ruled surface whose parametric vector equation, referred to asymptotic parameters u, v, is

$$x = x(u, v). \tag{1}$$

The four coordinates x of a variable point x on the surface satisfy two partial differential equations which can be reduced, by a suitably chosen transformation of proportionality factor, to Fubini's canonical form

$$x_{uu} = px + \theta_u x_u + \beta x_v, \qquad x_{vv} = qx + \gamma x_u + \theta_v x_v, \qquad \theta = \log \beta\gamma, \tag{2}$$

in which the subscripts indicate partial differentiation. The coefficients of these equations are functions of u, v and satisfy three integrability conditions which need not be written here.