FOURTEEN SPECIES OF SKEW HEXAGONS

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1. **Hexagon and hexahedron.** For a tentative definition, let a skew hexagon be a succession of six line segments or edges, finite or infinite, the terminal point of each being also the initial point of another, the terminus of the last being the first initial point. Secondly, let there let be six marks to designate the points, the six to be in a fixed order. Further, let the first point be any one of the six, so that only the cyclic order is essential. At each of the six points, pass a plane through the two lines, and denote the plane by the same mark as that point. Adjoin to the figure these six entire planes, a complete hexahedron.

2. **Sets of 64 hexagons.** The initial and terminal points of an edge denote ambiguously two segments, one finite and one infinite. If the marks in their order are 123456, then that cycle may denote equally any one of 64 different hexagons, $2^6=64$, obtained by selection. If desired, the one whose edges are all finite may be taken to represent the set. But we can define positive and negative directions along each line, arbitrarily on one, then by inference on all other lines of intersection of the planes. Thereafter it will be natural to understand by 12 a positive segment on the line, by 21 a negative.

3. **Hexagons and cyclic permutations.** As above defined, every skew hexagon determines one and only one set of six planes, but these intersect, not in 6 but in 15 lines, and so contain many different hexagons. Any cyclic permutation of the marks of the planes, as 154263, may be interpreted as describing a hexagon. An edge is given by any four consecutive marks; for example, 2631 will denote a segment on the intersection-line 63, from its point in plane 2 to its point in plane 1. Thus a cycle reversed will denote the same edges in reverse order and we shall consider the two as identical. As there are 120 cyclic permutations of 123456, this gives us only 60 different hexagons to assort into species.

All sets of 6 planes will be defined as equivalent, provided no four have a common point. By moving planes freely any one can be made to coincide with any other set, without passing through the excluded positions. It is well known that the traces of five planes in a sixth bound in that sixth one convex pentagon as well as certain triangles and quadrilaterals. Of the six pentagons, each has two edges which are edges of other pentagons, thus providing a natural order of the planes. Moreover those six common edges form a continuous