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ON THE DEFINITION OF CONTACT TRANSFORMATIONS

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If z is a function of $x_1, \cdots, x_n$ and $p_\nu = \partial z/\partial x_\nu$, $\nu = 1, \cdots, n$, a *contact transformation* in the space of $z, x_1, \cdots, x_n$, is defined by a set of $n+1$ equations

$$\text{(a)} \qquad Z = Z(z, x_\mu, p_\mu), \qquad X_\nu = X_\nu(z, x_\mu, p_\mu), \qquad \nu = 1, \cdots, n,$$

such that *firstly* in calculating the n derivatives

$$P_\nu = \frac{\partial Z}{\partial X_\nu}, \qquad \nu = 1, \cdots, n,$$

the expressions for the P_ν are given by a set of n equations

$$\text{(b)} \qquad P_\nu = P_\nu(z, x_\mu, p_\mu), \qquad \nu = 1, \cdots, n,$$

in which the derivatives of the p_μ *fall out*; and *secondly* the equations (a) and (b) can be resolved with respect to z, x_μ, p_μ:

$$\text{(A)} \qquad z = z(Z, X_\mu, P_\mu), \qquad x_\nu = x_\nu(Z, X_\mu, P_\mu), \qquad \nu = 1, \cdots, n,$$

$$\text{(B)} \qquad p_\nu = p_\nu(Z, X_\mu, P_\mu), \qquad \nu = 1, \cdots, n.$$

These two postulates are equivalent with the hypothesis that the $2n+1$ equations (a), (b) form a transformation between the two spaces of the sets of $2n+1$ independent variables (z, x_ν, p_ν), (Z, X_ν, P_ν) satisfying the Pfaffian condition

$$dZ - \sum_{\nu=1}^{n} P_\nu dX_\nu = \rho\left(dz - \sum_{\nu=1}^{n} p_\nu dx_\nu\right), \qquad \rho \neq 0.$$