A FIXED-POINT THEOREM FOR TREES[1]

A. D. WALLACE

By a *tree* we mean a compact (=bicompact) Hausdorff space which is acyclic in the sense that

(i) *if* $\mathfrak{U}$ *is a f.o.c.* (*=finite open covering*) *of a tree* T *then there is a f.o.c.* $\mathfrak{V}\subset\mathfrak{U}$ *such that the nerve* $N(\mathfrak{V})$ *is a combinatorial tree,*

and which is locally connected in the sense that

(ii) *if* $\mathfrak{U}$ *is a f.o.c. of* T *then there is a f.o.c.* $\mathfrak{V}\subset\mathfrak{U}$ *whose vertices are connected sets.*

It may be shown [3] that an acyclic continuous curve in the usual sense is a tree in our terminology. If q is a mapping which assigns to each point t of a topological space a *set* qt in a topological space, then we say that q is *continuous* provided that for each t and each neighborhood U of qt we can find an open set V containing t such that if t' is in V then qt' is in U. Our present purpose is to establish the following result:

(A) *Let* T *be a tree and let* q *be a continuous point-to-set mapping which assigns to each point* t *a continuum* qt *in* T. *Then there is a* $t_0\in T$ *such that* $t_0\in qt_0$.

The proof (which is divided into several lemmas) uses strongly a technique introduced by H. Hopf [1]. However the present note has been made self-contained.

(A_1) *The intersection of two continua of* T *is again a continuum.*

PROOF. Let B_1, B_2 be two continua such that $B_1\cdot B_2=C_1+C_2$ where the C_i are disjoint and closed. We can find disjoint open sets $D_i\supset C_i$. Let $t\in T-B_1\cdot B_2$. We can then find an open set V_t containing t and which does not meet both B_1 and B_2. The sets D_i together with the sets V_t can be reduced to a f.o.c. $\mathfrak{U}$ of T. Let $\mathfrak{V}\subset\mathfrak{U}$ be the f.o.c. described in (i). Let $\mathfrak{V}_i$ be those vertices of $\mathfrak{V}$ on B_i. It is easy to see that $N(\mathfrak{V}_i)$ is connected. If $c_j\in C_j$ we can find a chain of 1-cells E_i in $N(\mathfrak{V}_i)$ whose first vertex contains c_1 and whose last vertex contains c_2. Now we cannot have $E_i\subset D_1+D_2$ and E_i contains a vertex which is not on B_j. Hence $E_1\neq E_2$ and so $N(\mathfrak{V})$ is not a tree. This contradiction completes the proof.

[1] Presented to the Society, May 3, 1941.