author, upon carefully re-examining these paradoxes, has reached the following conclusions: (a) None of them arises if the would-be paradox maker is estopped from employing reasoning that may in fairness be rejected by people of sound understanding. (b) A conception of set may be delineated which accords with natural expectations and by means of which we may build a reliable theory of sets comprehending virtually all the results that have been subjected to attack. The relation is indicated which this conception bears to the basic conceptions, in the matter at issue, of the intuitionists (Brouwer and his school), the formalists (Hilbert and his school), the logicians (Russell and others), and the postulationists (Zermelo, Fraenkel). (Received July 29, 1941.)

430. A. R. Schweitzer: *Concerning general abstract relational spaces. III.*

On the basis of the general abstract relational space $S_{n+1}(G, H)$ $(n=1, 2, 3, \cdots)$, $G=H=$symmetric group on $n+1$ variables and certain axioms elaborating this space, the author constructs axioms for the (finite) algebra of logic analogous to his system ${}^{n+1}K_{n+1}$ for the foundations of geometry. For $n=3$ the elements of $S_4$ are $\alpha$, $\beta$, $\gamma$, $\delta$, $\lambda$, $\mu$, $\nu$, $\omega$ with $\alpha\beta\gamma\delta\ K$, that is, $\alpha\beta\gamma\delta\ K\ \alpha\beta\gamma\delta$. Axioms elaborating $S_4$ are the following: 1. $\alpha\beta\gamma\delta\ K \supset \lambda\mu\nu\omega\ K$. 2. $\alpha\beta\gamma\delta\ K$ and $\xi \supset \xi\beta\gamma\delta\ K$ or $\alpha\xi\gamma\delta\ K$ or $\alpha\beta\xi\delta\ K$ or $\alpha\beta\gamma\xi\ K$. 3. $\alpha\beta\gamma\delta\ K$ and $\lambda\mu\nu\omega\ K$ and $\xi$, $\eta \supset \alpha\lambda\xi\eta$ not $K$, $\beta\mu\xi\eta$ not $K$, $\gamma\nu\xi\eta$ not $K$, $\delta\omega\xi\eta$ not $K$. The complete set of $2^4$ $K$ tetrads is expressed as a reflexive formal sum $\sum(I)$ and classified into subsums: $\sum(\xi)$ is the sum of all $K$ tetrads containing $\xi$, and so on. If $\xi\eta\zeta\tau\ K$, then $\sum(\xi\eta\zeta\tau)=\xi\eta\zeta\tau$. The existence of a unique "empty" sum $\sum(0)=\sum(\alpha\lambda)=\sum(\beta\mu)=\sum(\gamma\nu)=\sum(\delta\omega)$ is assumed. The summands of the various $\sum$'s are replaced by their corresponding expressions in terms of $\sum$ and the $\sum$'s are then represented as products $\sum(\xi\eta)=\sum(\xi)\times\sum(\eta)$, and so on. The preceding continues a paper reported in this Bulletin (abstract 46-9-438). (Received July 22, 1941.)

## Statistics and Probability

431. K. J. Arnold: *On spherical probability distributions.*

Two methods of correspondence for circular distributions to the normal error function have led to non-constant absolutely continuous functions (see F. Zernike, *Handbuch der Physik*, vol. 3, pp. 477–478). The corresponding distributions for the sphere are found. The case of diametrical symmetry for both circle and sphere is discussed. Tables of the probability integrals involved are given and an application in geology is included. (Received July 31, 1941.)

432. I. W. Burr: *Cumulative frequency functions.*

Frequency and probability functions play a fundamental role in statistical theory and practice. They are, however, often inconvenient and difficult to use, since it is necessary to integrate or sum to find the probability for a given range. Theoretically the cumulative or integral frequency function would seem to be better adapted to determining such probabilities, since the latter can be found simply by a subtraction. The aim of this paper is to make a contribution toward the direct use of cumulative frequency functions. Some general properties and theory of cumulative functions are presented with particular emphasis upon certain moment functions adapted to such direct use. Both continuous and discrete cases are included. A list of possible cumulative functions is given and a particular one, $F(x)=1-(1+x^c)^{-k-1}$, discussed fully. This function has properties which make it practicable and adaptable to a wide variety