

lem is thereby reduced to one of the determination of branch points of a system of nonlinear equations by means of a characteristic value problem for a system of linearized equations. The relations between  $\delta M$ ,  $\delta N$  and  $\delta r$  are, for an elastic shell, those following from Hooke's law and from Navier's hypothesis. The complete system of scalar equations of this problem is obtained by a method analogous to that employed by the author in his simplified derivation of the equations for small displacements of thin shells (American Journal of Mathematics, vol. 63 (1941), pp. 177-184). (Received July 28, 1941.)

416. Alexander Weinstein: *On the buckling of a rectangular clamped plate compressed in one direction.*

This problem can be solved by an extension of a general method of reduction of eigenvalue problems (A. Weinstein, Mémorial des Sciences Mathématiques, vol. 88 (1937)). It can be linked to the corresponding problem for a supported plate by a sequence of intermediate differential problems of the fourth order which give lower bounds for the eigenvalues. It follows from this method that the lowest eigenvalue for a square plate of area  $\pi^2$  is  $>10.0$  while an upper bound 10.4 has been computed by J. L. Maulbetsch (Journal of Applied Mechanics, vol. 49 (1937)) who used the Rayleigh-Ritz method. The present method which gives definitely lower bounds for all eigenvalues differs essentially from previous formal procedures which could not establish such results. (Received July 18, 1941.)

#### GEOMETRY

417. E. F. Allen: *On a triangle inscribed in a rectangular hyperbola.*

In the study of inversive geometry the following formulas:  $z\bar{z}=a^2$ ,  $a^2z+t_1t_2\bar{z}=a^2(t_1+t_2)$ ,  $a^2z+t_1^2\bar{z}=2a^2t_1$ ,  $\bar{p}z+p\bar{z}=2a^2$  are respectively the self-conjugate equation of a circle, the equation of a line through two points on the circle, the equation of the tangent line, and the equation of the polar line of the point  $p$  with respect to the circle, where  $z=x+iy$ ,  $\bar{z}=x-iy$ , and  $i$  is defined by the equation  $i^2=-1$ . If a point in the  $xy$ -plane is designated by  $z=x+ry$ ,  $\bar{z}=x-ry$ , and  $r$  is defined by the equation  $r^2=+1$ , the base  $z\bar{z}=a^2$  is the rectangular hyperbola  $x^2-y^2=a^2$ . It is proved that the above formulas hold. They still hold if  $r$  is defined by  $r^2=-k$  or  $r^2=+k$ , where  $k$  is a real number. For any triangle inscribed in a rectangular hyperbola there exists a nine-point hyperbola having many of the characteristics of the nine-point circle of a triangle. An anti-orthocentric group of triangles is defined and it is proved that the four triangles of the group have a common nine-point hyperbola. (Received July 17, 1941.)

418. E. F. Beckenbach: *On the analytic prolongation of a minimal surface.*

In extension of a known result concerning the interior behavior of minimal surfaces, it is shown that if a minimal surface is bounded in part by a plane curve and if the surface approaches the plane orthogonally, then the surface can be extended analytically across the plane and the plane is a plane of symmetry for the extended minimal surface. (Received August 2, 1941.)

419. Nathaniel Coburn: *Semi-analytic unitary subspaces of unitary spaces.*

Suppose a Hermitian space  $X_m$  of  $m$ -dimensions is imbedded in an  $n$ -dimensional