

$\sum_{i=1}^n a_i^2 + r \sum_{i=1}^n b_i^2$  ( $n$  any positive integer,  $r=1, 2, 3$ ) are obtained. Other values of  $r$  can be treated by extending the function  $G(x, t)$ . This has been done for  $r=4, 5, 7$ , but the expansion for general  $r$  has not been obtained. The method also has been used to treat certain related forms. (Received July 29, 1941.)

361. R. D. James: *On the sieve method of Viggo Brun.*

A. A. Buchstab (Matematicheskii Sbornik, vol. 46 (1938), pp. 375–387, and Comptes Rendus de l'Académie des Sciences, URSS, vol. 29 (1940), pp. 544–548) has made improvements in the sieve method and so obtained more precise results. It is the purpose of this paper to point out that his method applies equally well to any infinite set of primes satisfying certain conditions. By applying the method to the set of primes congruent to 3 (mod 4), for example, it is shown that there is an infinite number of integer pairs  $n, n+4$ , each having exactly one prime factor congruent to 3 (mod 4). (Received July 28, 1941.)

362. Ivan Niven: *Quadratic diophantine equations in the rational and quadratic fields.*

Consider the general quadratic equation in two variables with rational integral coefficients, with non-negative discriminant (this restriction being imposed in order that the graph of the equation be not restricted to a finite region of the plane). Then one solution in integers of this equation implies infinitely many such solutions if and only if the graph of the equation is not an hyperbola with rational asymptotes or a pair of essentially irrational straight lines. If the coefficients of the equation are integers of a real quadratic field, one solution in integers of the field implies an infinite number if and only if the equation does not represent one of the following: no locus, a point, a pair of straight lines with coefficients essentially outside the quadratic field, or an ellipse with totally negative discriminant. A similar theorem is obtained for imaginary quadratic fields, the results being similar to the rational case. The principal parts of these theorems are proved by use of criteria for the solvability of the Pell equation in rational and quadratic fields; the criteria for quadratic fields are determined with the help of a theorem of Hilbert on the units of algebraic fields. (Received July 28, 1941.)

#### ANALYSIS

363. R. P. Agnew: *Analytic extension by Hausdorff methods.*

Let  $\chi(t)$ ,  $0 \leq t \leq 1$ , be a mass function which generates a regular Hausdorff method of summability  $H(\chi)$ . Let the number  $r$ , which is the greatest lower bound of numbers  $\rho$  such that  $\chi(t)$  is constant over  $\rho \leq t \leq 1$ , be called the *order* of  $H(\chi)$ . Let  $\sum c_n z^n$  be a power series with a positive finite radius of convergence. Corresponding to each vertex  $\zeta$  of the Mittag-Leffler star, let  $B(r, \zeta)$  denote the set of points  $z$  for which  $|z - (1 - r^{-1})\zeta| < r^{-1}|\zeta|$ . Let  $B(r)$  denote the set of inner points of the intersection of the sets  $B(r, \zeta)$ . It is shown that  $\sum c_n z^n$  is uniformly summable  $H(\chi)$  over each closed subset  $F$  of  $B(r)$ . The geometric series  $\sum z^n$  is non-summable  $H(\chi)$  at each point exterior to the closure of  $B(r)$ . A series  $\sum u_n$  is called summable  $\mathcal{H}$  to  $\sigma$  if there is at least one regular method  $H(\chi)$  which evaluates  $\sum u_n$  to  $\sigma$ . Some properties of the method  $\mathcal{H}$  are obtained. The existence of series with bounded partial sums which are not summable  $\mathcal{H}$  is implied by the following Tauberian gap theorem. If  $0 < n_1 < n_2 < \dots$ , if  $n_{p+1}/n_p \rightarrow \infty$  as  $p \rightarrow \infty$ , if  $u_n = 0$  when  $n \neq n_1, n_2, \dots$ , and if  $\sum u_n$  is summable  $\mathcal{H}$ , then  $\sum u_n$  is convergent. (Received June 23, 1941.)