SCALAR EXTENSIONS OF ALGEBRAS WITH EXPONENT EQUAL TO INDEX[1]

SAM PERLIS

If a normal simple algebra A has a special structural property, it is of interest to inquire whether every finite scalar extension A_K also has this property. We shall make such an inquiry here where the property assumed for A is equality of exponent and index.

It suffices to consider only separable and purely inseparable fields K. Roughly stated, our result for separable extensions of finite degree is that preservation of the property in question depends only on whether it is preserved for scalar extension fields which are cyclic of prime degree. In the case of purely inseparable extensions the problem is immediately reducible to the case of such extensions of prime degree p, where p is the characteristic of the field. There remains the question whether such extensions always preserve equality of exponent and index, and we shall answer this question in the negative by means of an example.

Every p-algebra over a field of degree of imperfection[2] unity has equal index and exponent.[3] The example mentioned above, however, shows that for every integer $r>1$ there exists a modular field of degree of imperfection r such that not all the p-algebras ($p=2$) over this field have exponent equal to index.

1. **Exponent reduction factor.** If A is any normal simple algebra of exponent ρ over F, the exponent of any scalar extension A_K is a divisor σ of $\rho=\sigma\tau$. The integer τ may be called the *exponent reduction factor* of A relative to K. This concept is analogous to that of index reduction factor and gives rise to a theorem analogous to that for index reduction factors.

THEOREM 1. *Let A be a normal simple algebra over F, and K be an algebraic extension of degree q over F. Then the exponent reduction factor of A relative to K is a divisor of q.*

PROOF. The direct power A^σ has exponent τ and K as splitting field. Now τ divides the index μ of A^σ, and μ divides the degree q of the splitting field K. Hence τ divides q.

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[2] For the concept of degree of imperfection see §3 of O. Teichmuller, *p-Algebren*, Deutsche Mathematik, vol. 1 (1936), pp. 362–388.

[3] Cf. O. Teichmuller, op. cit., p. 384. See also A. A. Albert, *p-algebras over a field generated by one indeterminate*, this Bulletin, vol. 43 (1937), pp. 733–736.