A NOTE ON THE SPECIAL LINEAR HOMOGENEOUS GROUP $SLH(2, p^n)$

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1. **Introduction.** The following theorem is due to E. H. Moore.

The special linear homogeneous group $SLH(2, p^n)$ of binary linear substitutions of determinant unity in the $GF[p^n]$ is simply isomorphic with the abstract group L generated by the operators T and S_λ, where λ runs through the series of p^n marks of the field, subject to the generational relations

(a) $S_0 = I$, $S_\lambda S_\mu = S_{\lambda+\mu}$ (λ, μ *any marks*),

(b) $T^4 = I$, $S_\lambda T^2 = T^2 S_\lambda$,

(c) $S_\lambda T S_\mu T S_{(1-\lambda)/(1-\lambda\mu)} T S_{1-\lambda\mu} T S_{(1-\mu)/(1-\lambda\mu)} T = I$ (λ, μ *any marks*, $\lambda\mu \neq 1$).

For $\lambda = 1$, $\mu \neq 1$, (c) gives

(d) $(S_1 T^3)^3 = I$.

Other relations employed by Dickson[1] in a proof of this theorem are

(e) $T S_\alpha T S_{2\alpha^{-1}} T S_\alpha T S_{2\alpha^{-1}} T^2 = I$ ($\alpha \neq 0$),

(f) $T S_\alpha T S_{\alpha^{-1}} T S_\rho = S_{\alpha^{-2}\rho} T S_\alpha T S_{\alpha^{-1}} T$ (ρ any mark).

It is the purpose of this paper to prove that (a), (b), (d), and (e) define an abstract group simply isomorphic with $SLH(2, p^n)$ when $p > 2$. If $p = 2$, relation (e) reduces to an identity and must be replaced by (f).

2. **Preliminary relations.** We first prove that (f) is a consequence of (a), (b), (d), and (e) when $p > 2$, so that in what follows we may use (f) for any p. We write (e) in the form

(e′) $T S_\alpha T = S_{-2\alpha^{-1}} T S_{-\alpha} T S_{-2\alpha^{-1}} T^2$

and make an even number of applications of this formula to the right member of (f) as follows:

$$S_{\alpha^{-2}\rho} \cdot T S_\alpha T \cdot S_{\alpha^{-1}} T = S_{\alpha^{-2}\rho - 2\alpha^{-1}} T S_{-\alpha} \cdot T S_{-\alpha^{-1}} T \cdot T^2$$

$$= S_{\alpha^{-2}\rho - 2\alpha^{-1}} \cdot T S_\alpha T \cdot S_{\alpha^{-1}} T S_{2\alpha} = S_{\alpha^{-2}\rho - 4\alpha^{-1}} T S_{-\alpha} \cdot T S_{-\alpha^{-1}} T \cdot S_{2\alpha} T^2$$

$$= S_{\alpha^{-2}\rho - 4\alpha^{-1}} \cdot T S_\alpha T \cdot S_{\alpha^{-1}} T S_{4\alpha} = S_{\alpha^{-2}\rho - 6\alpha^{-1}} T S_{-\alpha} \cdot T S_{-\alpha^{-1}} T \cdot S_{4\alpha} T^2$$

$$= S_{\alpha^{-2}\rho - 6\alpha^{-1}} \cdot T S_\alpha T \cdot S_{\alpha^{-1}} T S_{6\alpha} = \cdots = S_{\alpha^{-2}\rho - 2m\alpha^{-1}} \cdot T S_\alpha T \cdot S_{\alpha^{-1}} T S_{2m\alpha}.$$

Relation (f) is established by taking $m = \rho/2\alpha$. It will be convenient to write (f) in the equivalent form

(f′) $S_\rho T S_\alpha T S_{\alpha^{-1}} T = T S_\alpha T S_{\alpha^{-1}} T S_{\rho\alpha^2}$.

[1] *Linear Groups*, Leipzig, 1901. The notation is that employed by Dickson.