THE R_λ-CORRESPONDENT OF THE TANGENT TO AN ARBITRARY CURVE OF A NON-RULED SURFACE

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In a recent paper[1] the author defined at a general point y of a non-ruled analytic surface S the tangent line which he calls the *R_λ-correspondent* of the tangent at y to a general curve C_λ of S. It was proved[2] that (i) *a curve C_λ is a curve of Darboux if and only if at each of its points the R_λ-correspondent of the tangent to C_λ coincides with this tangent,* (ii) *a curve C_λ is a curve of Segre if and only if at each of its points the tangent to C_λ and its R_λ-correspondent are conjugate tangents of S.*

The primary purpose of this note is to present the following simple construction for the R_λ-correspondent: Let Λ denote a point of C_λ distinct from y, let U, V denote, respectively, the points of intersection of the asymptotic u- and v-curves passing through y with the asymptotic v- and u-curves passing through Λ, and let W denote the point of intersection of the tangent plane to S at y with the line joining the points U, V. *If y is held fixed while Λ tends toward y along C_λ, the point W describes a curve C_w and, except when C_λ is a curve of Segre or is tangent at y to a curve of Segre, the limit of W is the point y. The tangent at y to C_w is the R_λ-correspondent of the tangent to C_λ at y.*

The validity of this construction will be proved, and in addition the following theorem will be demonstrated:

A curve C_λ is a curve of Segre if and only if for a general point y of C_λ the limit of W as Λ tends to y along C_λ is a point W_0 distinct from y. The point W_0 is the intersection of the directrix of the first kind of Wilczynski with the tangent at y to the corresponding curve $C_{-\lambda}$ of Darboux.

Let the homogeneous projective coordinates $y^{(1)}, \cdots, y^{(4)}$ of a general point y on a non-ruled analytic surface S in ordinary space be functions of asymptotic parameters u, v. The functions $y^{(i)}$ are solutions of a system of differential equations, which can be reduced by a suitable transformation to Wilczynski's canonical form

$$y_{uu} + 2by_v + fy = 0, \qquad y_{vv} + 2a'y_u + gy = 0. \tag{1}$$

The coefficients of these equations are functions of u, v which are connected by three conditions of integrability. Moreover, the coordinates

[1] P. O. Bell, *A study of curved surfaces by means of certain associated ruled surfaces.* Transactions of this Society, vol. 46 (1939), pp. 389–409.

[2] Loc. cit., p. 393.