The corresponding expression for what I call the type A derivative—based on another, but equally logical definition—is merely the first term of the above expression.

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ON THE ASYMPTOTIC LINES OF A RULED SURFACE

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Many mathematicians have studied the surfaces every *asymptotic curve* of which belongs to a linear complex. I will here be content with the results given on pages 112–116 and 266–288 of a treatise[1] written by myself and Professor A. Cech. This treatise gives (p. 113) a very simple proof of the following theorem:

If every non-rectilinear asymptotic curve of a ruled surface S belongs to a linear complex, all these asymptotic curves are projective to each other.

We will find all the ruled surfaces, the non-rectilinear asymptotic curves of which *are projective to each other*, and prove conversely that *every one of these asymptotic curves belongs to a linear complex*. If c, c' are two of these asymptotic curves and if A is an arbitrary point of c, we can find on c' a point A' such that the straight line AA' is a straight generatrix of S. The projectivity, which, according to our hypothesis, transforms c into c', will carry A into a point A_1 of c'. We will prove that *the two points A' and A_1 are identical*; but this theorem is not obvious and therefore our demonstration cannot be very simple. The generalization to nonruled surfaces seems to be rather complicated: and we do not occupy ourselves here with such a generalization.

If the point $x=x(u, v)$ generates a ruled surface S, for which $u=$const. and $v=$const. are asymptotic curves, we can suppose (loc. cit., p. 182)

$$x = y + uz \tag{1}$$

in which y and z are functions of v. More clearly, if x_1, x_2, x_3, x_4 are homogeneous projective coordinates of a point of S, we can find eight functions y_i and z_i of v such that

$$x_i = y_i(v) + uz_i(v), \qquad i = 1, 2, 3, 4. \tag{1_{bis}}$$

From the general theory of surfaces, it is known (loc. cit., p. 90) that

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[1] *Geometria Proiettiva Differenziale*, Bologna, Zanichelli.