214. C. D. Olds: *On some arithmetical identities.*

In this note the author considers certain arithmetical identities of the type: $\sum \cos [2(a+b)\pi/p] = \sum \cos [2(a-b)\pi/p]$, $b \neq a$, where a and b range successively over all the quadratic residues of the given prime number p. Some applications are indicated. (Received March 8, 1941.)

215. Max Zorn: *Transcendental p-adic numbers related to roots of unity.*

Elementary remarks about some sequences which converge p-adically. (Received March 10, 1941.)

ANALYSIS

216. E. F. Beckenbach: *On functions having subharmonic logarithms.*

Two new characterizations of functions having subharmonic logarithms are given. One is expressed in function-theoretic terms, while the other is given by a generalized isoperimetric inequality. (Received March 10, 1941.)

217. Stefan Bergman: *The method of the minimum integral and the analytic continuation of functions.*

The measures of geometrical objects which occur in the theory of CT's (conformal transformations) and PT's (pseudo-conformal transformations) can be expressed as functions of minima $\lambda_{\mathfrak{B}}(t)$ of integrals $\int_{\mathfrak{B}} |h|^2 dx_1 dy_1 \cdots dx_n dy_n$, $z_k = x_k + iy_k$, where h runs through all functions analytic in $\mathfrak{B}$ and subjected to certain conditions at the point (t). If $\mathfrak{S} \subset \mathfrak{B}$ then $\lambda_{\mathfrak{S}}(t) \leqq \lambda_{\mathfrak{B}}(t)$, and since the equation $ds^2_{\mathfrak{B}}(z) = |dz|^2/\lambda_{\mathfrak{B}}(t)$ defines a metric invariant with respect to CT's the lemma of Schwarz-Pick (see Comptes Rendus de l'Académie des Sciences de l'URSS, vol. 16 (1937), p. 11) is obtained. On the other hand $\lambda_{\mathfrak{B}}(t)$ can be expressed with the aid of a system of functions orthogonal in $\mathfrak{B}$. Thus, in the case of Schlicht CT's one obtains certain refinements of the lemma of Schwarz-Pick. The introduction of the concept of B-area enables the author to generalize this technique to PT's. Finally, if one supposes that functions $w_k(z_1, z_2)$, $k=1, 2$, of the PT satisfy certain conditions on a surface bounding a segment $\mathfrak{a}$ of the boundary then w_1, w_2 are regular in $\mathfrak{a}$, and the PT (w_1, w_2) can be extended analytically through $\mathfrak{a}$ outside its original domain of definition. Applying then the method of the minimum integral for the extended domain, the author obtains results concerning distortion on the boundary for PT's. (Received April 1, 1941.)

218. Lipman Bers: *On a generalized harmonic measure.*

Suppose D is a domain in R_n, B its (bounded) boundary, B' the set of the regular boundary points of D, $u(P)$ $(P \in D)$ a bounded harmonic function (b.h.f.). There exists a function $\mu_u(P, e)$ (generalized harmonic measure) which is a b.h.f. of P for every fixed Borelian set $e \subset B$, a completely additive set function for every fixed point $P \in D$ and satisfies the condition $\lim [\mu_u(P, e) - u(P)] = 0$, $\lim \mu_u(P, B-e) = 0$ $(P \to R \in B')$ for every open set $e \subset B$, $R \in e$. If $f(Q)$ is a continuous function, $v(P) = \int_B f(Q) d\mu_u(P, e_Q)$ is the solution of the following problem (a generalization of the Dirichlet problem): determine a b.h.f. v such that $\lim [v(P) - f(R)u(P)] = 0$ $(P \to R \in B')$. If f is a bounded Baire function, v is a b.h.f. which is "representable by u." The bounded harmonic functions which possess a positive lower bound and which