ON A PROPERTY OF k CONSECUTIVE INTEGERS[1]

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S. S. Pillai[2] has just proved the following theorem: In every set of less than 17 consecutive integers there exists at least one integer which is relatively prime to all the others; there are sequences of k integers for $k=17, 18, \cdots, 430$, however, which have not this property. Pillai conjectures that the same is valid for every $k \geqq 17$. I shall prove that this conjecture is true.

The method of the proof is similar to the method I applied in a joint paper with H. Zeitz[3] in proving that the following conjecture is wrong for every prime $p \geqq 43$.

Denote by p_n the nth prime. Then there exist at most $2p_{n-1}-1$ consecutive integers such that each of these integers is at least divisible by one of the primes $p_1, p_2, \cdots, p_n$.

This conjecture was used by Legendre for his proof of the theorem of the primes in arithmetical progressions. First I prove the following.

LEMMA. *Let $\pi(x)$ be the number of primes $p \leqq x$. Then we have*

$$\pi(2x) - \pi(x) \geqq 2\left[\frac{\log x}{\log 2}\right] + 2 \tag{1}$$

for every $x \geqq 75$.

PROOF. If we put, as usual,

$$\vartheta(x) = \sum_{p \leqq x} \log p,$$

then we have

$$\begin{aligned} \pi(2x) - \pi(x) &= \sum_{x<p\leqq 2x} 1 \geqq \sum_{x<p\leqq 2x} (\log p/\log 2x) \\ &= \left\{ \sum_{x<p\leqq 2x} \log p \right\} /\log 2x = \left\{\vartheta(2x) - \vartheta(x)\right\}/\log 2x. \end{aligned} \tag{2}$$

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[1] Presented to the Society, September 12, 1940.

[2] S. S. Pillai, *On m consecutive integers*, Proceedings of the Indian Academy of Sciences, section A, vol. 11 (1940), pp. 6–12.

[3] A. Brauer und H. Zeitz, *Über eine zahlentheoretische Behauptung von Legendre*, Sitzungsberichte der Berliner mathematischen Gesellschaft, vol. 29 (1930), pp. 116–125. Cf. A. Brauer, *Question concerning the maximum term in the diatomic series—proposed by A. A. Bennett*, American Mathematical Monthly, vol. 40 (1933), pp. 409–410.