NON-INVOLUTORIAL SPACE TRANSFORMATIONS ASSOCIATED WITH A $Q_{1,2}$ CONGRUENCE

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De Paolis[1] discussed the involutorial transformations associated with the congruence of lines meeting a curve of order m and an $(m-1)$-fold secant, while Vogt[2] studied the transformation T for a linear congruence and bundle of lines. In the present paper the transformations associated with the congruence of lines on a conic and a secant of it are discussed.

Given a conic r, a line s meeting r once, and two projective pencils of surfaces

$$|F_{n+m+1}| : \quad r^n s^m g; \qquad |F'_{n'+m'+1}| : \quad r^{n'} s^{m'} g',$$

where $n \leqq m+1$, $n' \leqq m'+1$, $[r, s]=A$, and g, g' the residual base curves.

Through a generic point P, there passes a single surface F of $|F|$. The unique line t through P, r, s meets the associated F' in one residual point P', image (T) of P. The transformations to be considered are of three types:

Case I. $n=m+1$, $n'=m'+1$.

Case II. $n<m+1$, $n'<m'+1$.

Case III. $n=m+1$, $n'<m'+1$.

CASE I

Given

$$|F_{2n}| : \quad r^n s^{n-1} g; \qquad |F'_{2n'}| : \quad r^{n'} s^{n'-1} g';$$

where g, g' are of order n^2+2n-1, $n'^2+2n'-1$. The curve g meets r, s in n^2+2n-1, n^2-1 points respectively.

The conic r is a fundamental curve whose image (T^{-1}) is R: $r^{n+n'}$, since there are $(n+n')$ invariant directions through each point on r. R is generated by a monoidal plane curve of order $n+n'+1$, one curve on each plane of the pencil $(O_r s)=w$, as O_r describes r. The fundamental line s has for image (T^{-1}) a surface S: $s^{n+n'-1}$, of which $n+n'-2$ branches are invariant. A is a fundamental point of the first kind, whose image (T^{-1}) is the plane $u: r$. In the plane $v: s$ and tangent

[1] De Paolis, *Alcuni particolari trasformazioni involutori dello spazio*, Rendiconti dell' Accademia dei Lincei, Rome, (4), vol. 1 (1885), pp. 735–742, 754–758.

[2] Vogt, *Zentrale und windschiefe Raum-Verwandtschaften*, Jahresbericht der Schlesischen Gesellschaft für Vaterländische Kultur, class 84, 1906, pp. 8–16.