COMMENTS ON CANONICAL LINES

R. B. RASMUSEN AND B. L. HAGEN

1. **Introduction.** In this paper we propose to find the equations of the canonical edges of Green using a conjugate net as the parametric net of an analytic surface; to give a new interpretation to the line called by Davis the associate line of collineation; and, finally, to make a generalization of the canonical quadric of Davis.

2. **Analytic basis.** Let the projective homogeneous coordinates $x^1, \cdots, x^4$ of a point P_x on a surface S in ordinary space be analytic functions of two independent variables u, v. In the notation[1] of Lane if the parametric curves on S form a conjugate net, the coordinates x of the point P_x and the coordinates y of a point which is the harmonic conjugate of the point P_x with respect to the foci of the axis of the point P_x, satisfy a system of equations of the form

$$\begin{aligned} x_{uu} &= px + \alpha x_u + Ly, \\ x_{uv} &= cx + ax_u + bx_v, \\ x_{vv} &= qx + \delta x_v + Ny, \qquad LN \neq 0. \end{aligned} \tag{1}$$

The ray-points of the net at the point P_x are given by the formulas

$$x_{-1} = x_u - bx, \qquad x_1 = x_v - ax.$$

Some of the invariants of the net are

$$\begin{aligned} H &= c + ab - a_u, & K &= c + ab - b_v, \\ \mathfrak{H} &= c + ab + b_v - \delta_u, & \mathfrak{K} &= c + ab + a_u - \alpha_v, \\ 8\mathfrak{B}' &= 4a - 2\delta + (\log r)_v, & r &= N/L, \\ 8\mathfrak{C}' &= 4b - 2\alpha - (\log r)_u. \end{aligned} \tag{2}$$

If the covariant tetrahedron, x, x_1, x_{-1}, y is used as the local tetrahedron of reference, a power series expansion[2] for one nonhomogeneous coordinate z of a point on the surface in terms of the other two coordinates x, y is

$$\begin{aligned} z = {} & \tfrac{1}{2}(Lx^2 + Ny^2) + \tfrac{4}{3}(L\mathfrak{C}'x^3 + N\mathfrak{B}y^3) + c_0x^4 \\ & + 4c_1x^3y + 4c_3xy^3 + c_4y^4 + \cdots, \end{aligned} \tag{3}$$

[1] Lane, *Conjugate nets and the lines of curvature*, American Journal of Mathematics, vol. 53 (1931), p. 574.

[2] Lane, *A canonical power series expansion for a surface*, Transactions of this Society, vol. 37 (1935), p. 481.