NOTE ON PROBABILITY IMPLICATION

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In a recently published paper[1] J. C. C. McKinsey has pointed out some difficulties which arise from Axiom I of my theory of probability implication.[2] This axiom states the unambiguity of the degree p of a given probability implication $(O \ni_p P)$ for the case that the class O is not empty, a condition formulated by $(\overline{\overline{O}})$, but postulates ambiguity of p in case of an empty class O, this condition being formulated by $(\overline{O})$. The latter ambiguity is necessary for probability implication because of the relation to Russell's material implication.[3] From the proof published by McKinsey we can infer that this ambiguity has to be restricted to values of p between 0 and 1, limits included, in correspondence with the same restriction holding for the unambiguous degree p of probability in cases of a non-empty class O, formulated by me in (8, §13).[4] That this general restriction is derivable from Axiom II, 2 is obvious as this axiom contains O and p as free variables and therefore states the restriction for all classes O and all values p.

A further objection, which was already indicated in a footnote of McKinsey's paper, has been presented to me in a letter by the referee of this journal, Mr. S. C. Kleene. This objection shows that if the ambiguity of degrees of probability for empty classes O is assumed, it can be proved that this ambiguity cannot be restricted to the limits 0 to 1.

This proof is connected with the theorem of addition (Axiom III) which reads[5]

III. $(O \ni_p P).(O \ni_q Q).(O.P \supset \overline{Q}) \supset (\exists r)(O \ni_r P \vee Q).(r = p+q)$.

The condition $r \leqq 1$ implies that $p+q \leqq 1$. If we demand $r \leqq 1$ only for non-empty classes O, the mentioned restriction for p and q, which

[1] This Bulletin, vol. 45 (1939), pp. 799–800.

[2] Published in *Wahrscheinlichkeitslehre*, Leiden, 1935, §§12–14. My further quotations refer to this book.

[3] Page 66.

[4] To avoid misunderstandings let me add here the remark that this relation is meant only for the case that the probability $W(O, P)$ *exists*, and would be written in the implicational writing

$$[(\exists x)\ (O \ni_x P)] \supset [(\exists y)\ (O \ni_y P).(0 \leqq y \leqq 1)].$$

If O is not empty and therefore the probability has only one value, this means that this value is restricted to the limits 0 to 1, limits included.

[5] I write here the existential operator on the right-hand side because the abbreviation introduced on page 62, according to which the existential operator is omitted in the corresponding formula of my book, may be misleading.