SPACE CREMONA TRANSFORMATIONS OF ORDER $m+n-1$[1]

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1. **Introduction.** This paper discusses a space Cremona transformation of order $m+n-1$ (m, n any integers) generated by two rational twisted curves. One special position of the defining curves gives rise to an involution recently described,[2] while another special position results in an involution somewhat similar to one which was defined in a different manner by Montesano.[3]

2. **Cremona transformation.** Consider a curve C_n of order n having $n-1$ points on each of two skew lines d and d', and a curve C_m' of order m having $m-1$ points on each of d and d' (m, n, any integers). A generic point P determines a ray through it intersecting C_n once in α and d once in β. P also determines a ray through it intersecting C_m' once in γ and d once in δ. We define P', the correspondent of P, to be the intersection of lines $\alpha\delta$ and $\beta\gamma$.

It is to be noted that if C_n should become identical with C_m' but d and d' remain distinct, there would result the Cremona involution we discussed in a recent paper (loc. cit.).

Let the equations of d be $x_1=0$, $x_2=0$, and those of d' be $x_3=0$, $x_4=0$. Let C_n be

$$x_1 = (as + bt)\prod_1^{n-1} (t_is - s_it), \qquad x_2 = (cs + dt)\prod_1^{n-1} (t_is - s_it),$$

$$x_3 = (es + ft)\prod_n^{2n-2} (t_is - s_it), \qquad x_4 = (gs + ht)\prod_n^{2n-2} (t_is - s_it),$$

where s_i, t_i for $i=1, 2, \cdots, n-1$ are values of the parameters of C_n for points on d, and for $i=n, n+1, \cdots, 2n-2$, for points on d'.

Let the equations of C_m' be

$$x_1 = (AS + BT)\prod_1^{m-1} (T_iS - S_iT), \quad x_2 = (CS + DT)\prod_1^{m-1} (T_iS - S_iT),$$

$$x_3 = (ES + FT)\prod_m^{2m-2} (T_iS - S_iT), \quad x_4 = (GS + HT)\prod_m^{2m-2} (T_iS - S_iT),$$

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[2] E. J. Purcell, *A multiple null-correspondence and a space Cremona involution of order* $2n-1$, this Bulletin, vol. 46 (1940), pp. 339–444.

[3] D. Montesano, *Su una classe di trasformazioni involutorie dello spazio*, Rendiconti del' Istituto Lombardo di Scienze e Lettere, (2), vol. 21 (1888), pp. 688–690.