165. Peter Scherk: *On real closed curves of order $n+1$ in projective n-space.* II. Preliminary report.

In the first part of this paper (abstract 46-11-502) the author discussed differentiable closed curves K^{n+1} of real order $n+1$ in R_n by means of a certain single-valued correspondence of the K^{n+1}. He proved that $S \leqq n+1$, $S \equiv n+1 \pmod 2$ if S is the sum of the multiplicities of the singular points, and he characterized the case $S=n+1$. Extending a simple remark on rotation numbers to multi-valued correspondences, the author discusses a two-valued and a three-valued correspondence defined on certain arcs of the K^{n+1} and on the whole K^{n+1} respectively, and connected with the projections of the K^{n+1} from its osculating $(n-2)$-spaces and $(n-3)$-spaces respectively. The study of these two correspondences yields: (1) the first estimates of the number of osculating $(n-2)$-spaces which meet the K^{n+1} again; (2) the classification of the K^{n+1} with $S=n-1$; (3) the classification of the K^5; (4) a more systematic access to the classification of the K^4 (previously obtained by the author). (Received January 24, 1941.)

166. Alexander Wundheiler: *Abstract algebraic definition of an affine vector space.* Preliminary report.

A linear set over the field of real numbers will be called a simple vector space, and its elements, simple vectors. Two simple vector spaces A and B are cogrediently coupled if for any a in A and b in B a real number $f(a, b)$ is defined, such that $f(ka, b)=f(a, kb)=kf(a, b)$; $f(a, b'+b'')=f(a, b')+f(a, b'')$; $f(a'+a'', b)=f(a', b)+f(a'', b)$. The a's and b's are then contragredient vectors. If A and B are of the same dimension, the set $A+B$ is called an affine vector space, a is a contravariant affine vector, b a covariant one, or vice versa. Various illustrations are given, as electrical networks, the space of fruit juice cocktail cans, and so on. (Received January 24, 1941.)

167. Oscar Zariski: *Pencils on an algebraic variety and a new proof of a theorem of Bertini.*

The theorem of Bertini-Enriques states that if a linear system of W_{r-1}'s on a V_r is reducible (that is, every W_{r-1} of the system is reducible) and is free from fixed components, then the system is composite with a pencil. In this paper a new proof of this theorem is given, together with an extension to irrational pencils. With every pencil $\{W\}$ there is associated a field P of algebraic functions of one variable, a subfield of the field Σ of rational functions on V_r. The essential point of the proof is the remark that $\{W\}$ is composite if and only if P is not maximally algebraic in Σ. The rest of the proof, in the case of pencils, follows from the fact that an irreducible algebraic variety V_r over a ground field K is absolutely irreducible if K is maximally algebraic in Σ. In the case of linear systems of dimension >1, the proof is based on the following lemma: if K is maximally algebraic in Σ and if x_1, x_2 are algebraically independent elements of Σ, then for all but a finite number of elements c in K the field $K(x_1+cx_2)$ is maximally algebraic in Σ. (Received December 12, 1940.)

Logic and Foundations

168. Alvin Sugar: *Postulates for the calculus of binary relations in terms of a single operation.*

In a recent paper (*Postulates for the calculus of binary relations*, Journal of Symbolic Logic, vol. 5 (1940), pp. 85–97) J. C. C. McKinsey gave a set of postulates for the