ON A RESULT OF HUA FOR CUBIC POLYNOMIALS[1]

ALVIN SUGAR

In a paper by L. K. Hua,[2] we find the following principal result.

THEOREM. *For any positive integer* ϵ, *every integer can be expressed in infinitely many ways as a sum of seven values of the cubic function*

$$f(x) = \epsilon(x^3 - x)/6 + x$$

for integral values of x; *also, every integer can be expressed in infinitely many ways as a sum of seven values of*

$$F(x) = (x^3 - x)/6$$

for integral values of x.[3]

In this paper we get a better result by applying a known identity for cubes to generalizations of the above polynomials. We state our results in the following two theorems. Note that, in Theorem 1, ϵ may be positive or negative, or, for that matter, zero.

Unless otherwise stated all letters in this paper stand for integers, positive, negative, or zero.

THEOREM 1. *For any* ϵ, c, *and any* k *prime to* ϵ, *every integer can be expressed in infinitely many ways as a sum of five values of the function*

$$p(x) = \epsilon(x^3 - x)/6 + kx + c$$

for integral values of x.

THEOREM 2. *For any* k *and* c, *every integer can be expressed as a sum of four values of the function*

$$P(x) = (x^3 - x)/6 + kx + c$$

for integral values of x.

Theorem 1 is trivially true when $\epsilon = 0$. For, in this case, $(\epsilon, k) = 1$ implies $k = 1$. Henceforth we take $\epsilon \neq 0$.

---

[1] Presented to the Society, April 27, 1940.

[2] *On the representation of integers by the sums of seven cubic functions*, Tôhoku Mathematical Journal, vol. 41 (1935–1936), pp. 361–366.

[3] Hua fails to mention in his formulation of this theorem whether his ϵ may take negative values. It seems that Hua implicitly assumed ϵ positive, as was noted by Pall in his review of the Hua paper in the Zentralblatt für Mathematik, vol. 14, p. 10. (This assumption was probably unnecessary, however.)