The max $|l_1^{(n)}(x)|$ is attained at $x=\pm 1$ since[4] (I) $\theta_{k+1}-\theta_k \leqq 2\pi/(2n+\alpha+\beta-1)$ provided $\frac{1}{2}\leqq\alpha$, $\beta\leqq\frac{3}{2}$ and $x_k\equiv\cos\theta_k$. Using the second asymptotic formula and the fact[4] that $n\theta_k\to j_k$ as $n\to\infty$ where j_k is the kth positive zero of $J_{\beta-1}(x)$, we find that

$$|l_k^{(n)}(1)|\to\left(\tfrac{1}{2}j_k\right)^{\beta-2}|\Gamma(\beta)J_\beta(j_k)|^{-1} \qquad \text{as } n\to\infty,\ k \text{ constant},$$

$l_1^{(n)}(-1)\to 0$ which proves the theorem:

THEOREM 7. *Max* $|l_1^{(n)}(x)|\to(\frac{1}{2}j_1)^{\beta-2}|\Gamma(\beta)J_\beta(j_1)|^{-1}$ *as* $n\to\infty$ *(where* $\frac{1}{2}\leqq\alpha$, $\beta\leqq\frac{3}{2}$, j_1 *is first positive zero of* $J_{\beta-1}(x)$*).*

A similar result holds for $l_n^{(n)}(x)$ if β is replaced by α.

For Legendre polynomials ($\alpha=\beta=1$) this limit is approximately 1.602. For $\alpha=\beta=\frac{1}{2}$ and $\alpha=\beta=\frac{3}{2}$ the limit of Theorem 7 is also an upper bound for max $|l_1^{(n)}(x)|$ and max $|l_k^{(n)}(x)|$. Whether this is true, in general, remains unanswered.

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AN INVARIANCE THEOREM FOR SUBSETS OF S^n [1]

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The purpose of this paper is to establish the following.

INVARIANCE THEOREM. *Let A and B be two homeomorphic subsets of the n-sphere* S^n. *If the number of components of* S^n-A *is finite, then it is equal to the number of components of* S^n-B.

In the case when A and B are closed this theorem is a very well known consequence of Alexander's duality theorem and its generalizations. In our case we also derive our result as a consequence of a duality theorem. However, the duality is established only for the dimension $n-1$.

Given a metric space X we shall say that Γ^k is a k-cycle in X if there is a compact subset A of X such that Γ^k is a k-dimensional convergent (Vietoris) cycle in A with coefficients modulo 2. We shall write $\Gamma^k\sim 0$ if $\Gamma^k\sim 0$ holds in some compact subset of X. The homology group of X obtained this way will be denoted by $\mathcal{H}^k(X)$; the corresponding connectivity number, by $p^k(X)$. The number $p^k(X)$ can be either finite or ∞.

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[1] Presented to the Society, December 28, 1939.