of years from $I_n$ corresponding to $n$ years. Various assumptions regarding the frequency distribution of the precipitation lead to corresponding sets of theoretical factors for standardizing the precipitation ratio and certain of these agree well with observations. Results plotted on coordinate paper having a reciprocal scale for $I_n$ and a logarithmic scale for $n$ facilitate a graphical treatment of the observations since the graph approximates to a straight line. The standard error of $I_n$ is also computed corresponding to the two most important frequency distributions of the precipitation. In order to reduce the accidental fluctuations of $I_n$ it is sometimes desirable to use the difference between the averages of the $rn$ highest and the $rn$ lowest values, where $rn$ equals some appropriate value, say 3. Theoretical factors derived for correcting such values to obtain $I_n$ agree well with observations. (Received October 3, 1940.)

### 78. Jerzy Neyman: *A statistical problem in mass production and routine analyses.*

Let $x_{i1}, x_{i2}, \cdots, x_{in}$ denote the results of parallel analyses of some $i$th sample, $i=1, 2, \cdots, N$ (a sample of items of mass production). The $x$'s are considered as random variables following the law $p(x_{ij})=(\sigma_i(2\pi)^{1/2})^{-1}\exp\{(x_{ij}-\xi_i)^2/2\sigma^2_i\}$. Denote by $H$ the hypothesis that $\sigma_1=\sigma_2=\cdots=\sigma_N$, that is, that the precision of measuring $\xi_i$ is maximum, and let $x_i=(\sum_j x_{ij})/n$, $S_i^2=\sum_j(x_{ij}-x_i)^2$, $V=\sum_i S_i^2$, $U=\sum_i S_i^4$. Also let $E$ denote the observed point in the space $W$ of the $S_i^2$'s and $w(V)$ the part of the locus $V=$const. included in any region $w\subset W$. It is proved that (1) a necessary and sufficient condition for a test's probability of rejecting $H$ when true to be a fixed value $\alpha$ is that its critical region $w=\sum_v w(V)$, with the $w(V)$ arbitrary except that $P\{E\in w(V)\mid E\in W(V)\}=\alpha$ for any $V>0$. (2) If $H$ is not true and $h=\sigma^{-2}$ is a random variable with $p(h)=ch^{\alpha-1}e^{-\beta h}$, then the critical region $w_0$ determined by $U>U_\alpha V^2$, with $P\{U>U_\alpha V^2\mid H, V\}=\alpha$, has the following useful properties: (a) if $H$ be true, then the probability of $w_0$ rejecting $H$ is equal to $\alpha$, (b) the first derivative of the power function of $w_0$, taken at the point of $H$ being true, is equal to zero, (c) the second derivative is maximum. ((b) is true for all regions satisfying (a).) (Received October 28, 1940.)

## Theory of Numbers

### 79. E. T. Bell: *Selective equations.*

The symbols ( )′, [ ]′ denote the least, greatest of the integers occurring within the symbols. Each such integer may be replaced by a symbol ( )′, [ ]′ referring to a new set of integers, and so on, a finite number of times. The most general system of equations consisting of equalities between single power products formed from ( )′, [ ]′ is solved non-tentatively and finitely, by passing to a unique dual of the system, in which ( ), the G.C.D., and [ ], the L.C.M., replace ( )′, [ ]′. The latter system is a simple multiplicative system, and hence is completely solvable non-tentatively and finitely. The problem solved arose in the detailed discussion of compound multiplicative systems. (Received October 26, 1940.)

### 80. Leonard Carlitz: *Finite differences and polynomials in a Galois field.*

This paper is concerned chiefly with the solution of equations such as $\sum f(M)M(t)=g(t)$ and $\sum f(M, u)M(t)=g(u, t)$, the summation extending over all polynomials $M=M(x)$ of degree less than $m$. (Received November 25, 1940.)