

irreducible subspaces. This paper is concerned with a study of the law of combination satisfied by the indices of the tensor associated with each diagram or partition of the indices of the arbitrary tensor. It is noted that corresponding to each partition there can be associated a specialization of C. M. Cramlet's general invariant tensor (*Tôhoku Mathematical Journal*, vol. 28 (1927), pp. 242–250), and that an arbitrary tensor can be decomposed by multiplying an appropriate numerical tensor identity by it, and then contracting. (Received November 25, 1940.)

11. R. W. Wagner: *The differentials of analytic matrix functions.*

The notions of Hausdorff and Fréchet concerning differentials are applied to analytic matrix functions as defined by the author in a previous paper. The differential of an analytic matrix function is shown to have a simple form when it is written in terms of the characteristic roots and partial idempotent and partial nilpotent elements of the argument. The function enters the differential through divided difference quotients. This simple form of the differential is used to deduce local properties of the mapping defined by the function. (Received November 23, 1940.)

12. Morgan Ward: *The fundamental theorem of arithmetic.*

A set of necessary and sufficient conditions for the fundamental theorem of arithmetic to hold in a semi-group is obtained from the theory of residuated lattices and a new inductive proof of the theorem is given for the lattice of positive integers. (Received October 30, 1940.)

13. Hermann Weyl: *Theory of reduction for arithmetical equivalence.*
II.

Instead of the arithmetically refined method of reduction used in the first paper, the author now resorts to a rougher method which also goes back to Minkowski, and works without the assumption that the class number of ideals is 1. Its generalization to algebraic number fields F with several infinite prime spots is due to Siegel and P. Humbert (*Commentarii Mathematici Helvetici*, vol. 12 (1939–1940), pp. 263–306). The author follows the same geometric approach as before, including all classes of lattices over F and adding to the case of a field F that of a field quaternion algebra with totally positive norm over a totally real field. (Received November 23, 1940.)

ANALYSIS

14. Warren Ambrose: *Representation of ergodic flows.* Preliminary report.

A flow is a one-parameter group T_t ($-\infty < t < \infty$) of measure preserving transformations of a space Ω into itself. It is measurable if the function $T_t P$ is a measurable function on $T \times \Omega$, where T denotes the real line taken with Lebesgue measure. For a measurable ergodic flow it is shown that a necessary and sufficient condition that a group of unitary operators U_t defined by $U_t f(P) = f(T_t P)$ have an eigenvalue (other than the trivial eigenvalue 1) is that the flow be isomorphic (with respect to measure properties) to a flow built on a measure preserving transformation (for definition of such a flow see abstract 46-11-446), thus showing that if an ergodic flow has an eigenvalue the measure on Ω must be the direct product measure of a cross section measure with Lebesgue measure along the trajectories of the flow. It is also shown that if the flow has no eigenvalues but satisfies certain conditions which are stronger than