CONTINUA OF FINITE DEGREE AND CERTAIN PRODUCT SETS[1]

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The continua of finite degree have been studied and identified with certain well known classes of continua in a paper by G. T. Whyburn.[3] The author has shown that the continua of finite degree are identical with the continua homeomorphic to a continuum of finite length.[4] The object of the present note is to obtain other internal characterizations of these continua.

The symbol M will represent a (compact) continuum. The continuum M is said to be of finite degree at the point $p \in M$ provided that to each $\epsilon > 0$ there corresponds an *uncountable* family of neighborhoods (U) of p such that (a) the diameter of each neighborhood is less than ϵ, (b) each $F(U)$ is finite, where $F(U)$ is the boundary of U, and (c) for any pair of neighborhoods U and V either $\overline{U} \subset V$ or $\overline{V} \subset U$. If every point is of finite degree, the continuum M is said to be of finite degree. The characterization which we find most useful below is that a continuum M is of finite degree if and only if every subcontinuum contains uncountably many local separating points of M.

It will be shown that the classes of continua defined by each of the following properties are identical with the continua of finite degree.

PROPERTY N^0. *M is locally connected and to each pair of closed, disjoint subsets A and B in M there corresponds a finite collection of disjoint, perfect sets $H^1, H^2, \cdots, H^k$ such that any continuum K in M intersecting both A and B contains some H^i.*

PROPERTY Q. *If K and K_i, $(i=1, 2, \cdots)$, are nondegenerate continua in M with $\lim K_i = K$, there exists an integer n such that $\prod_n^\infty K_i$ is an uncountable set.*

It will be noted that the Property N^0 is highly analogous to Property N which characterizes the locally connected continua such that no true cyclic element has a continuum of condensation.[5]

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[3] G. T. Whyburn, *Concerning continua of finite degree and local separating points*, American Journal of Mathematics, vol. 57, pp. 11–16.

[4] See abstract 45-9-321, this Bulletin.

[5] This concept is due to R. L. Moore. See his *Fundamental Point Set Theorems*, Rice Institute Pamphlets, vol. 23, no. 1, 1936.

For this characterization see G. T. Whyburn, *On continua of condensation*, American Journal of Mathematics, vol. 58, pp. 705–708.