THE BASES OF PROBABILITY[1]

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The subject for consideration today forms an aspect of a somewhat venerable branch of mathematical theory; but in essence it is part of a far older department of thought—the ancient science of logic. For it is concerned with a category of propositions of a nature marked by features neither physical nor mathematical, but by their rôle under the aspect of the reason. Their essential characteristic is their involvement of that species of relation between the knower and the known evoked by such terms as probability, likelihood, degree of certainty, as used in the parlance of intuitive thought. It is our threefold task to transcribe this concept into symbols, to formulate its principles, and to study its properties in their inner order and outward application.

As prelude to this undertaking it is necessary to set forth certain conventions of logic. Propositions are the elements of symbolic logic, but they may play the rôle of *contemplated propositions* (statements in quotation marks) or of *asserted propositions* (statements regarded as true throughout a given manipulation or deduction); and it is necessary to take account of this in the notation for the logical constants. We shall employ the symbols for negation ($\sim$), conjunction or logical product ($\cdot$), and disjunction or logical sum ($\vee$), and regard them as having no assertive power: they combine contemplated propositions into contemplated propositions and asserted propositions into asserted propositions *of the same logical type*. Quite other shall be our convention regarding implication ($\subset$) and equivalence ($=$): they combine contemplated propositions into asserted propositions, and shall not be used to combine asserted propositions in our present study. If a and b stand for contemplated propositions, the assertion that a is false (true) shall be written $a=0$ ($a=1$), and the assertion that a implies b, $a\subset b$ or $a\sim b=0$; it is thus quite different from the contemplated proposition $\sim(a\sim b)$. Finally it is universally asserted that $a\sim a=0$, $a\vee\sim a=1$; and in fact all the laws of Boolean algebra are regarded as assertions. We shall assume their elements to be fa-

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For the details of the theory here expounded, see the two publications of the present author. *The axioms and algebra of intuitive probability*, Annals of Mathematics, (2), vol. 41 (1940), pp. 269–292 (herein to be abbreviated as AAP) and *Intuitive probability and sequences* (forthcoming in the Annals of Mathematics) (abbreviation PS).