THE MINIMAL NUMBERS OF BINARY FORMS[1]

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1. **Introduction.** One of us proved that for certain fields K a form F of degree m can be written as a linear combination of mth powers of linear forms. Such a combination is termed a *representation* of F and the least possible number of terms in any such representation is called the *minimal* number of F with respect to K. The minimal number depends on both F and K. For fields K with characteristic greater than n, and binary forms F of degree n, it has been proved[2] that the minimal number ranges over at least $1, 2, \cdots, n$, and at most $1, 2, \cdots, n+1$, but the exact range was not determined. In the present paper the authors prove that the range is precisely $1, 2, \cdots, n$.

2. **Preliminary lemmas.** In what follows we use *identity* of polynomials in the usual sense, namely polynomials P and Q are identical if the coefficients of P equal the corresponding coefficients of Q.

Since the order of a field K is greater than m if the characteristic of K is greater than m, we have the following lemma.

LEMMA 1. *For a field K with characteristic greater than m a polynomial P of degree m is equal to a polynomial Q for all values of the variables if and only if P and Q are identical.*

An immediate consequence of Lemma 1 is the following lemma.

LEMMA 2. *For a field K with characteristic greater than m, a polynomial P of degree m not identically zero is different from zero for at least one set of values of the variables.*

LEMMA 3. *Let K be a field with characteristic greater than m. Let Δ be the determinant*

$$\Delta = \begin{vmatrix} 1 & \cdots & 1 & b_1 \\ a_1 & \cdots & a_m & b_2 \\ \cdot & \cdots & \cdot & \cdot \\ a_1^m & \cdots & a_m^m & b_{m+1} \end{vmatrix} \tag{1}$$

of order $m+1$, $m \geqq 1$, with elements in K, and suppose that the b's are not all zero. The determinant Δ is not identically zero in the a's.

[1] Presented to the Society, April 13, 1940.

[2] R. Oldenburger, *Polynomials in several variables*, Annals of Mathematics, (2), vol. 41 (1940), no. 3, pp. 694–710.