THE SOLUTION OF BOUNDARY VALUE PROBLEMS BY A DOUBLE LAPLACE TRANSFORMATION

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1. **Introduction.** The Laplace transformation has most frequently been used to transform a linear partial differential equation with two independent variables and constant coefficients into an ordinary differential equation, the "subsidiary equation," from the solution of which that of the original equation is deduced. If it is applied to an equation with more than two independent variables, the subsidiary equation is also a partial differential equation and is usually solved by classical methods. The object of this note is to point out that partial differential equations in which the range of two (or more) independent variables is $(0, \infty)$ may easily be handled by simultaneous Laplace transformations in these variables. The point of view is that of previous papers[1] in which the Laplace transformation method has been regarded as purely formal, and the solution as subject to verification. The method is related to that of Doetsch[2] for equations of elliptic type but assumes no theoretical basis. In §3 a simple two variable problem is discussed to illustrate the method and in §§4 and 5 are given two new three variable problems of a type to which it is well adapted.

2. **The general method.** We consider a function $v(x, t)$ in the range $x>0$, $t>0$.

Let

$$u_0(x) = v(x, 0), \qquad u_1(x) = \left[\frac{\partial v}{\partial t}\right]_{t=0},$$

$$w_0(t) = v(0, t), \qquad w_1(t) = \left[\frac{\partial v}{\partial x}\right]_{x=0}.$$

The Laplace transform with respect to t will be denoted by a "bar," thus

$$\bar{f}(p) = \int_0^\infty e^{-pt} f(t)\,dt,$$

and that with respect to x by a capital letter, thus

[1] Carslaw and Jaeger, Proceedings of the Cambridge Philosophical Society, vol. 35 (1939), p. 394; Proceedings of the London Mathematical Society, in press; this Bulletin, vol. 45 (1939), p. 407.

[2] Doetsch, *Theorie und Anwendungen der Laplace-Transformation*, Berlin, 1937, chap. 22.