ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

353. Stefan Bergman: *On a generalized Green's function and certain of its applications.*

A theorem of Blaschke states: $\sum_{\nu=1}^{\infty} \lg |a_\nu| > -\infty$, $|a_\nu| < 1$, is the necessary and sufficient condition for the existence of a nonnegative harmonic function $H(z)$, $H(0) < \infty$, $z \,\varepsilon\, \mathfrak{E}^2 - \mathfrak{S}_\nu\{a_\nu\}$, $\mathfrak{E}^2 = \mathrm{E}[|z| < 1]$ which possesses the property that $H(z) + \log|z - a_\nu|$, $\nu = 1, 2, \cdots$, is regular at $z = a_\nu$. Let $\mathfrak{M}^4 = \mathrm{E}[|z_2| < 1, z_1 \,\varepsilon\, \mathfrak{B}^2(z_2)]$, $\mathfrak{B}^2(z_2) = \mathrm{E}[z_1 = sh(z_2, \lambda), 0 \leqq s < 1, 0 \leqq \lambda \leqq 2\pi]$, $h(z_2, \lambda)$, $h(z_2, 0) = h(z_2, 2\pi)$, being for every λ an analytic function of z_2, $z_2 \,\varepsilon\, \overline{\mathfrak{E}}^2$. In the paper (Compositio Mathematica, vol. 6 (1939), pp. 307–335) there was introduced for every domain $\mathfrak{M}^4$ an "extended class of functions" possessing properties analogous to those of harmonic functions. If $g(z_1, z_2)$ is in $\mathfrak{M}^4$ an analytic function, it is then possible to construct the "Green's function of the extended class" $\Gamma(z_1, z_2; g; \mathfrak{M}^4)$ vanishing on the boundary surface $\mathrm{E}[|z_2| = 1, z_1 = h(z_2, \lambda), 0 \leqq \lambda \leqq 2\pi]$ such that $\Gamma(z_1, z_2; g; \mathfrak{M}^4) + \lg|g(z_1, z_2)|$ is a function of extended class. Let $g_\nu(z_1, z_2)$ designate a set of functions analytic in $\mathfrak{M}^4$ satisfying a certain additional restriction concerning the distribution of zero surfaces. The condition $\sum_{\nu=1}^{\infty} \Gamma(0, 0; g_\nu; \mathfrak{M}^4) < \infty$ is necessary and sufficient for the existence of a nonnegative function $H(z_1, z_2)$, $H(z_1, z_2) < \infty$ for $(z_1, z_2) \,\varepsilon\, \mathfrak{M}^4 - \mathfrak{S}_\nu \mathfrak{G}_\nu^2$, $\mathfrak{G}_\nu^2 = \mathrm{E}[g_\nu(z_1, z_2) = 0, (z_1, z_2) \,\varepsilon\, \mathfrak{M}^4]$ such that for every n, $H(z_1, z_2) + \sum_{\nu=1}^{n} \lg |g_\nu(z_1, z_2)|$, belongs to the extended class of functions of $\mathfrak{M}_{n0}^4$, $n_0 \leqq n$, where $\lim_{n\to\infty} \mathfrak{M}_n^4 = \mathfrak{M}^4$. (Received March 28, 1940.)

354. L. M. Blumenthal: *A new concept in distance geometry, with applications to spherical subsets.*

If $\sigma \subset S$, a semimetric space, then S has σ-relative congruence indices $\{n, k\}$ with respect to a given class (Σ) of semimetric spaces provided any space Σ of (Σ) with more than $n+k$ pairwise distinct points is congruently contained in S whenever each n of its points is congruently contained in σ. This concept gives rise to new problems in distance geometry. For $\sigma = S$ the indices $\{n, k\}$ are called merely congruence indices of S with respect to (Σ). The notions of congruence and quasi congruence orders introduced by Menger correspond to congruence indices $\{n, 0\}$, $\{n, 1\}$, respectively, while $\{n, 2\}$ signifies a property weaker than quasi congruence order n but stronger than quasi congruence order $n+1$. Let $\kappa_{2,\rho}$ denote a cap with spherical radius ρ of the sphere $S_{2,r}$. It is shown that (1) if $\sigma = \kappa_{2,\pi r/2}$, with base circle removed, then $S_{2,r}$ has σ-relative congruence indices $\{4, 2\}$ with respect to all semimetric spaces (Σ); (2) the closed cap $\kappa_{2,\rho}$, $\rho < \pi r/2$, has congruence indices $\{4, 2\}$ with respect to (Σ); (3) $\kappa_{2,\rho}$, $\rho < r \cos^{-1} 1/3$, has indices $\{4, 1\}$. Analogous theorems for caps of $S_{n,r}$ are obtained. (Received April 12, 1940.)