

acquaint the reader with a skeleton of methods such as he is apt to encounter in the calculus of variations, rather than to formulate results as generally as possible. The first chapter is devoted to a discussion of various problems, and the derivation of the first order necessary conditions for an extremum. Chapter II treats quadratic problems; in particular, the theory of integral equations with real symmetric kernel and boundary value problems associated with a second order linear differential equation. Chapter III, which deals with sufficient conditions, is extremely brief; nowhere in the lectures is the Weierstrass  $\mathcal{E}$ -function mentioned. In Chapter IV the work of Lewy (*Mathematische Annalen*, vol. 98 (1928), pp. 107–124) on the absolute minimum for nonparametric problems in the plane is presented. Chapter V is concerned with harmonic functions and associated boundary value problems; this chapter is preliminary to the discussion of the problem of Plateau and conformal mapping given in Chapter VI. In this last chapter the problem of Plateau is solved for a single contour in three dimensions; the method of proof is that of Courant (*Annals of Mathematics*, (2), vol. 38 (1937), pp. 676–724).

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*Die Zylinderfunktionen und ihre Anwendungen.* By R. Weyrich. Leipzig, Teubner, 1937. 5+137 pp.

A cylinder function may be defined as any solution of Bessel's differential equation. They include functions of the first kind,  $J_p(z)$ , also called Bessel functions, which are regular at  $z=0$ , and functions of the second kind which are not regular at  $z=0$ . There has been considerable confusion in the notation and canonical form of the cylinder functions of the second kind. They have been denoted by  $Y$ ,  $G$ ,  $K$ , and so on, by various authors and often the same notation is used with different meanings. Following the tables of Jahnke and Emde, our author uses the notation  $N_p(z)$  for functions of the second kind and calls them Neumann functions. They are the same as those denoted by  $Y_p(z)$  by Nielsen and also by Watson. (See Watson, *Theory of Bessel functions*, p. 57, for a further discussion.) In addition there are the Hankel cylinder functions, which are really functions of the second kind, defined by

$$H_p^{(m)}(z) = J_p(z) + i(-1)^{m+1}N_p(z), \quad m = 1, 2, i = (-1)^{1/2}.$$

The importance of the Hankel functions arises from the fact that alone among the cylinder functions  $H_p^{(m)}(re^{i\theta}) \rightarrow 0$  as  $r \rightarrow \infty$ , provided that  $m = 1$ ,  $0 \leq \theta \leq \pi$ , and that  $m = 2$ ,  $\pi \leq \theta \leq 2\pi$ .

One way to introduce cylinder functions is to define them, as above,