SIMPLICIAL INTERSECTION CHAINS FOR AN ABSTRACT COMPLEX

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Abstract complexes have been defined by J. W. Alexander, W. Mayer, A. W. Tucker and S. Lefschetz.[1] The definition here adopted is that of Lefschetz which derives directly from that of Tucker. To conform to Lefschetz's present usage the notation here will differ from that in the article cited by having $K=\{E_i^p\}$ and $K^*=\{E_p^i\}$. It should be remarked that when K is infinite $FF=0$ implies that for given E_i^p and E_j^{p-2},

$$[E_i^p:E_k^{p-1}][E_k^{p-1}:E_j^{p-2}]\neq 0$$

for at most a finite number of E_k^{p-1}. The dual K^* here is to have the property

$$[E_i^p:E_j^{p-1}] = (-1)^{p+1}[E_{p-1}^j:E_p^i] = \mu_{ij}^p$$

where μ_{ij}^p is an abbreviated notation (compare loc. cit., p. 346 (c)). This is in order that a fourth postulate

IV. Ki $(E_i^p\cdot E_p^i)=1$

may be added to I–III (loc. cit., p. 350), where Ki means Kronecker index, the sum of the coefficients of the elements of the zero-chain in question. Here C is a p-chain of K, D a q-chain of K^*, $p>q$.

It is well known that when K is a simplicial or polyhedral manifold, for every pair E_i^p, E_q^j, $p\geqq q$, an intersection can be defined which is a chain of the simplicial regular subdivision of K. And when K is an arbitrary simplicial complex, the intersection is on K and hence simplicial. Here it is shown that *for any complex the intersections* $E_i^p\cdot E_q^j$ *can be regarded as integral chains of a simplicial complex* K^π.

This complex will be defined abstractly by means of its vertices σ_i^p. The first definition is for $p=q$:

$$E_i^p\cdot E_p^j = \delta_i^j\sigma_j^p \tag{1}$$

(where $\cdot$ replaces the $\odot$ of the article cited). Then for $p>q$, if $r=p-q-1$,

$$E_i^p\cdot E_q^j = \sum_{k_1,k_2,\cdots,k_r} \mu_{ik_1}^p\mu_{k_1k_2}^{p-1}\cdots\mu_{k_rj}^{q+1}\sigma_i^p\sigma_{k_1}^{p-1}\cdots\sigma_{k_r}^{q+1}\sigma_j^q \tag{2}$$

[1] S. Lefschetz, this Bulletin, vol. 43 (1937), pp. 345–359. (References to the other authors will be found on page 345.)