CERTAIN SELF-RECIPROCAL FUNCTIONS

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In 1932 and 1933 I [5, 6] gave some rules connecting different classes of self-reciprocal functions. The object of this note is to derive some new self-reciprocal functions with the help of those rules.

I will say that a function is R_ν if it is self-reciprocal for J_ν transforms, where $\nu > -1$.

I will make use of the following results given in the papers referred to:

If $f(x)$ is R_μ, the functions $g(x)$ given by the following integral formulas are all R_ν:

$$\text{(i)} \qquad g(x) = x^{(\nu-\mu+1)/2} \int_0^\infty y^{(\nu-\mu+1)/2} J_{(\mu+\nu)/2}(xy) f(y)\,dy,$$

$$\text{(ii)} \qquad g(x) = x^{(\mu-\nu+1)/2} \int_0^\infty y^{(\mu-\nu+1)/2} J_{(\mu+\nu)/2}(xy) f(y)\,dy,$$

$$\text{(iii)} \qquad g(x) = \int_0^\infty \frac{y^{\mu+1/2} f(xy)}{(1+y^2)^{1+\mu/2+\nu/2}}\,dy,$$

$$\text{(iv)} \qquad g(x) = \int_1^\infty \frac{y^{1/2-\mu} f(xy)}{(y^2-1)^{1-\mu/2+\nu/2}}\,dy,$$

$$\text{(v)} \qquad g(x) = \int_0^1 \frac{y^{1/2+\mu} f(xy)}{(1-y^2)^{1+\mu/2-\nu/2}}\,dy.$$

If, in (ii) we take the familiar R_μ function

$$\text{(1)} \qquad x^{\mu+1/2} e^{-x^2/2}$$

for $f(x)$, we get

$$g(x) = x^{(\mu-\nu+1)/2} \int_0^\infty y^{(\mu-\nu+1)/2} J_{(\mu+\nu)/2}(xy) \cdot y^{\mu+1/2} e^{-y^2/2}\,dy$$

$$= x^{(\mu-\nu+1)/2} \int_0^\infty y^{3\mu/2-\nu/2+1} e^{-y^2/2} J_{\mu/2+\nu/2}(xy)\,dy.$$

Evaluating this integral by Hankel's formula [7], we get

$$g(x) = x^{(\mu-\nu+1)/2} \frac{\Gamma(\mu+1)(x/2^{1/2})^{\mu/2+\nu/2}}{2^{-3\mu/4+\nu/4}\Gamma(1+\mu/2+\nu/2)} e^{-x^2/2} \cdot {}_1F_1(\nu/2-\mu/2;\, 1+\mu/2+\nu/2;\, x^2/2), \qquad \mu > -1.$$