calculus, which can only explain its rise and growth in the sixteenth and seventeenth centuries, still has to be written.

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Algebren. By M. Deuring. (Ergebnisse der Mathematik, vol. 4, no. 1.) *Gruppen von linearen Transformationen*. By B. L. van der Waerden (Ergebnisse der Mathematik, vol. 4, no. 2.) Berlin, Springer, 1935. 5+143 and 3+91 pp., respectively.

The theory of algebras, now about to enter the second century of its existence, constitutes today an integrating part of algebra and arithmetics. The most fundamental step in its development seems to have been the introduction of general reference fields, essentially due to Wedderburn. In order to describe approximately the degree of generality we may say that Wedderburn's theory holds at least for those fields which obey the theory of Galois. We quote from Dickson's *Linear Algebras* (1914): "Any linear associative algebra over a field F is the sum of a semisimple algebra and a nilpotent invariant subalgebra (the radical) each over F. A semisimple algebra is either simple or the direct sum of algebras over F. Any simple algebra over F is the direct product of a division algebra and a simple matrix algebra each over F."

Other results are found in Dickson's book *Algebras and their Arithmetics* (1923), which concludes with an instructive list of unsolved problems: (I) the determination of all division algebras, (II) the classification of nilpotent algebras, the discovery of relations between an algebra and its maximal nilpotent invariant subalgebra (the radical), (III) theory of non-associative algebras, and (IV) theory of ideals in the arithmetic of a division algebra and the extension to algebras of the whole theory of algebraic numbers.

Progress in the study of problems II and III has been moderate, in the sense that we have many beautiful special results but no general theory.

As to problems I and IV, our knowledge has advanced considerably, to say the least; this advance is reported in Deuring's report *Algebren* and Albert's Colloquium lectures *Structure of Algebras*. We quote both authors in saying that R. Brauer, H. Hasse, E. Noether and A. A. Albert have given the solution of problem I for algebraic number fields. From the authors who contributed to the solution of problem IV we single out (at the expense of others) H. Brandt who provided the fundamental idea that a left ideal in one maximal domain of integrity is a right ideal in another, and his pupil Eichler