

ANOMALOUS PLANE CURVE SYSTEMS ASSOCIATED WITH SINGULAR SURFACES¹

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1. **Introduction.** A plane curve system of order n and genus p , with δ nodes, κ cusps, ι statangents, τ bitangents, is of first virtual dimension d_0 , of virtual dimension d'_0 , and of effective dimension d . For such a system, $d'_0 = 3n + p - \kappa - 1 = 3m + p - \iota - 1$. If the curve system has only distinct nodes and cusps, $d_0 = d'_0$; if higher singularities, $d_0 < d'_0$.

For any irreducible, continuous, complete curve system, $d \geq d_0$. If $d = d_0$, the curve system is called homalious, if $d > d_0$, anomalous. The anomaly A of a plane curve system is defined by the relation $A = d - d_0$. The above definitions were introduced by B. Segre.²

The sections by a plane π of the tangent cones to a nonsingular surface of order ν from the points of S_3 constitute a continuous plane curve system with the characteristics:

$$\begin{aligned} n &= \nu(\nu - 1), & \kappa &= \nu(\nu - 1)(\nu - 2), \\ \delta &= (1/2)\nu(\nu - 1)(\nu - 2)(\nu - 3). \end{aligned}$$

B. Segre³ has shown that for this curve system,

$$\begin{aligned} d_0 &= (5/2)\nu(\nu - 1), & d &= (1/6)(\nu + 1)(\nu + 2)(\nu + 3) - 5, \\ A &= d - d_0 = (1/6)(\nu - 2)(\nu - 3)(\nu - 4). \end{aligned}$$

Hence, for $\nu \geq 5$, this plane curve system is anomalous.

The purpose of the present paper is to ascertain the dimensions and anomaly of plane sections of tangent cones to certain singular surfaces.

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² B. Segre, *Esistenza e dimensione di sistemi continui distinti di curve piane algebriche con dati caratteri*, Rendiconti dell'Accademia dei Lincei, (6), vol. 10 (1929), pp. 31-38. The adjective "irregolare" used by B. Segre to describe the plane curve system for which $d > d_0$ has two English equivalents "irregular" and "anomalous." The first translation is the more natural one. Since it has been found, however, that similar systems of surfaces exist and since the term "irregular" has long had a different definite meaning for surfaces, another term is necessary for surfaces and, therefore, should also be used for plane curves. Homalious and anomalous are Greek antonyms. Only the latter has heretofore been used in English.

³ B. Segre, *Sulla caratterizzazione delle curve di diramazione dei piani multipli generali*, Memorie delle Reale Accademia d'Italia, Classe di Scienze Fisiche, Matematiche e Naturali, vol. 1 (1930), pp. 5-31.