

## BOOK REVIEWS

*Elementary Number Theory.* By J. V. Uspensky and M. A. Heaslet. New York and London, McGraw-Hill, 1939. 10+484 pp.

Numerous historical references and applications of the theory presented, as well as detailed proofs, make this book especially suited to novices in the field. As noted in the introduction "owing to self-imposed limitations in the size of the book, many topics of interest had to be omitted." Analytical and geometrical methods have been avoided, while topics such as continued fractions and integral transformations of forms have been omitted entirely. On the other hand the book covers with thoroughness various fundamental problems which have inspired much of the research in this field. The results are illustrated by numerous examples, some solved in the text, others left to the reader.

The first five chapters are devoted to various problems which can be readily treated without the use of congruences. In fact, the notion of congruence is deferred until the sixth chapter when the reader has become familiar with a number of the fundamental concepts of number theory. The first chapter contains a discussion of elementary properties of integers, a method of computing polygonal numbers, and a discussion of scales of notation. It also contains an analysis, in terms of binary representations of numbers, of the generalized Chinese game of Nim. In the second chapter there is a treatment of common divisors and multiples, and a solution of the Diophantine equation  $x^2 + y^2 = z^2$  in integers. The third chapter contains Lamé's theorem, Euclid's least remainder algorithm, and application of this algorithm to the solution of linear Diophantine equations. The fourth chapter is devoted to a discussion of prime numbers. The topics considered are the sieve of Eratosthenes, the unique factorization theorem, the number and sum of divisors of an integer, perfect numbers, Mersennes's numbers, and the distribution of primes. In the fifth chapter there is a discussion of relative primeness, and in particular of Euler's function  $\phi(n)$  and Moebius's function  $\mu(n)$ , with results based on a well known combinatorial theorem. This theorem is applied to the problem of determining the number of primes less than a given integer, and the chapter closes with Meissel's formula for this number.

After the introduction of the concept of "congruence," Chapter 6 is devoted to Fermat's theorem concerning the congruence  $a^{p-1} \equiv 1 \pmod{p}$  and the Euler generalization on  $a^{\phi(m)} \equiv 1 \pmod{m}$ . Several