

WEIERSTRASS PREPARATION THEOREM

H. SERBIN

H. Späth¹ has shown that the Weierstrass preparation theorem is a consequence of the following one:

THEOREM. *Let P and Q be power series in the variables z_1, z_2, \dots, z_m such that P is regular in z_1 of degree k , that is, $P(z_1, 0, \dots, 0) = cz_1^k + dz_1^{k+1} + \dots$, ($c \neq 0$). Then there exist power series A and B such that*

$$(1) \quad Q - AP = B$$

where B does not contain powers of z_1 higher than the $(k-1)$ st. The series A and B are uniquely determined.

It has not been observed that this general form permits of a simple proof by induction when we are concerned only with *formal* series. The solution of (1) is equivalent to the solution of the system

$$(2) \quad \begin{aligned} (q_n - a_0 p_n - a_1 p_{n-1} - \dots - a_{n-1} p_1) - a_n p_0 &= b_n, \\ n &= 0, 1, 2, \dots, \end{aligned}$$

where $P = \sum p_n z_m^n$, $Q = \sum q_n z_m^n$, $A = \sum a_n z_m^n$, $B = \sum b_n z_m^n$. The theorem is evidently true for the case $m=1$. Since p_0 is regular in z_1 of degree k , the existence of a formal solution of (1) and its uniqueness is an immediate consequence of an induction from $m-1$ to m variables.

PRINCETON, N.J.

¹ H. Späth, *Journal für die reine und angewandte Mathematik*, vol. 161 (1929), pp. 95-100. This contains additional references.