SOME THEOREMS ON CONTINUA[1]

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The following theorem, proved by A. Mullikin in her thesis,[2] has been used extensively in certain point set theoretic investigations.

THEOREM. *If C is a continuum and F_1 and F_2 are closed, mutually exclusive and nonvacuous subsets of C, then there exists a component of $C-(F_1+F_2)$ which has a limit point in F_1 and a limit point*[3] *in F_2.*

In §1 we shall obtain a theorem (Theorem 1) which represents a strengthening of Mullikin's result. The theorem, in this stronger form, has numerous applications. We shall discuss some of these applications in §2 and §3.

1. **A stronger form of Mullikin's theorem.** We prove the following theorem:

THEOREM 1. *Under the hypotheses of Mullikin's theorem, there exists a constituent*[4] *of $C-(F_1+F_2)$ which has a limit point in F_1 and a limit point in F_2.*

PROOF. Let $\{G_n\}$ and $\{H_n\}$ denote monotonic decreasing sequences of open sets which close down upon F_1 and F_2 respectively. We may suppose that G_1 and H_1 are so chosen that $\overline{G}_1\cdot\overline{H}_1=0$. Now, by Mullikin's theorem, there is a connected subset of $C-C\cdot(\overline{G}_n+\overline{H}_n)$ which has a limit point in $\overline{G}_n$ and a limit point in $\overline{H}_n$. The closure of such a connected set is a subcontinuum of C which "extends" from $\overline{G}_n$ to $\overline{H}_n$. If, then, we denote by Q_n the set of all points of $C-C\cdot(G_n+H_n)$ which lie on subcontinua of C which extend (in the above indicated sense) from $\overline{G}_n$ to $\overline{H}_n$, we have $Q_n\neq 0$.

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[1] Presented to the Society, April 10, 1937, under the title *On a theorem due to A. Mullikin.*

[2] *Certain theorems relating to plane connected point sets,* Transactions of this Society, vol. 24 (1922), pp. 144–162.

[3] In Mullikin's statement of this theorem, C is a bounded plane continuum. It is clear, however, that her proof applies if C is any continuum in a compact metric space. In the present paper, *all point sets under consideration are understood to be embedded in a compact metric space,* except when the contrary is expressly stated. If M is a point set and p is a point of M, then the maximal connected subset of M which contains p is called the component of M determined by p.

[4] A set of points K is said to be strongly connected, or to be a semicontinuum, if every pair of points of K lies on some continuum contained in K. If M is a point set and p is a point of M, then the maximal strongly connected subset of M which contains p is called the constituent of M determined by p.