ON THE CONCEPT OF A RANDOM SEQUENCE[1]

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Von Mises has based his frequency theory of probability on the notion of a *Kollektiv*,[2] that is, of an infinite sequence of trials of an event whose possible outcomes have each a definite probability but otherwise appear entirely at random. (Convenient illustrative examples are an infinite sequence of tosses of a coin, an infinite sequence of rolls of a die, and the like.)[3]

Abstractly the *Kollektiv* may be represented by an infinite sequence of points of an appropriate space, the *Merkmalraum*. Or if the number of possible outcomes of a trial is finite (and it may well be argued that this is always the case for any actual physical observation[4]), it is sufficient to employ an infinite sequence of natural numbers which are less than a fixed natural number. This infinite sequence—of points or of natural numbers—satisfies certain conditions which correspond to those appearing in the description of a *Kollektiv* as just given, and which we shall express by saying that it is a random sequence (*regellose Folge*).

For the present purpose it is largely sufficient to confine attention to the case that each trial has only two possible outcomes, as with the toss of a coin adjudged as falling heads or tails, or the roll of a die adjudged as showing or not showing an ace. The *Kollektiv* may then be represented abstractly by a random sequence of 0's and 1's: in the case of the coin, for instance, we may let 1 correspond to the fall of heads and 0 to tails.

The definition of a random sequence of 0's and 1's as given by von Mises may perhaps be put in the following form:

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[3] The introduction of an *infinite* sequence of trials (tosses of a coin, and so on) is, of course, an abstraction from the realities of the situation, made for the sake of the mathematical theory. It is an instance of the familiar device of employing the infinite as being, for certain purposes, a convenient and useful approximation to the large finite.

[4] In cases where the number of possible outcomes of a trial is taken as infinite, either there is a further element of abstraction involved (the infinite again replacing the large finite), or else the problem considered has no direct physical application.