

BOOK REVIEWS

General Analysis. Part 2. The Fundamental Notions of General Analysis. By Eliakim Hastings Moore. (Memoirs of the American Philosophical Society, vol. 1, part 2.) Philadelphia, American Philosophical Society, 1939. 6+255 pp.

The first part of this work, embodying the contributions of the late E. H. Moore on general analysis, was reviewed in this Bulletin, vol. 42 (1936), pp. 465–468, by C. C. MacDuffee.¹ The first part was devoted in the main to considerations of an algebraical character, no notion of continuity or limit being involved. While a general variable appears in the considerations, its generality is only incidental in that it is usually permitted to assume at most a finite number of distinct values. This second part begins a study from the point of view of analysis, in that limits notions play a substantial role. It is general analysis in the sense that a general unconditioned variable is involved.

The basis of this part includes a number system \mathfrak{A} of the type used at the end of the first part, with a continuity axiom added, namely, the existence of a greatest lower bound for nonvacuous sets of positive numbers. The resulting number system is then shown to be isomorphic to either the real number system, the complex number system, or the system of quaternions. There is assumed a general range \mathfrak{B} unconditioned. Vectors enter as functions on \mathfrak{B} to \mathfrak{A} , matrices as functions on $\mathfrak{B}^1\mathfrak{B}^2$ to \mathfrak{A} , and so on. While recent developments in linear functional analysis usually postulate a vector without considering it as a function on some range, the assumption made here, aside from its inherent advantages, has additional justification in that virtually all examples of the notion of vector have the character of a function.

The first chapter of this part (Chapter IV of the complete work) is devoted to an exposition of the notion of general limit. The general limit is a natural generalization of the notion of limit of a sequence, the positive integers being replaced by a general set \mathfrak{L} of elements l , on which there is given a relation R on pairs of elements which is transitive, that is, such that $l_1 R l_2$ and $l_2 R l_3$ imply $l_1 R l_3$, and compositive or semiordered, that is, l_1 and l_2 imply the existence of an l

¹ A review of this same part by the present reviewer appeared in *Zentralblatt für Mathematik*, vol. 13 (1936), pp. 116–117. We take this opportunity to correct a misprint appearing in MacDuffee's review, namely, the definition of general reciprocal λ of a matrix κ in the second last line of page 467 should read $SS\lambda\kappa = \kappa$ instead of $SS\lambda\kappa = \kappa$.