THE ANALOGUE OF THE MOEBIUS GROUP OF CIRCULAR TRANSFORMATIONS IN THE KASNER PLANE*

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1. **Introduction.** We shall begin by giving some fundamental definitions. In this paper, by a curve of the plane π we shall mean a differential element of the third order in the plane π. A *simple horn-set* consists of all the curves (third order differential elements) in the plane π which possess a common point and a common direction. Let x denote the curvature and $y = dx/ds$ the rate of variation of the curvature per unit length of arc s of any curve of a simple horn-set at the common point. Then any curve of a simple horn-set is given by an ordered pair of numbers (x, y). From this, it follows that a simple horn-set of the plane π is a two-dimensional space, called the *Kasner plane* K_2, where any point of K_2 is a curve (x, y) of the simple horn-set. Thus to a given simple horn-set of the plane π, there is associated an auxiliary plane, called the Kasner plane K_2, such that any given point of the Kasner plane represents a curve of the simple horn-set whose curvature and rate of variation of the curvature per unit length of arc at the common point are the abscissa x and the ordinate y of the given point.

Kasner has shown that the group of conformal transformations in the plane π operating on the curves of a simple horn-set induces the *three-parameter group* G_3:

$$X = mx + h, \qquad Y = m^2y + k, \tag{1}$$

where $m \neq 0$, h, k are constants, from the points (x, y) to the points (X, Y) of the Kasner plane.† The Kasner plane is thus an affine plane.

A line consists of the ∞^1 points $C(x, y)$ of the Kasner plane which satisfy a linear equation in x and y with the coefficients of x and y not both zero. With respect to the group G_3, the lines of the Kasner plane may be classified into three distinct types.

(a) A *general line* is a line whose equation is of the form $y = px + r$, where $p \neq 0$ and r are constants.

(b) An *infinite line* is a line whose equation is of the special form $y = \text{const.}$

* Presented to the Society, February 25, 1939.

† Kasner, *Conformal geometry*, Proceedings of the Fifth International Congress of Mathematicians, Cambridge, 1912, vol. 2. Kasner and Comenetz, *Conformal geometry of horn angles*, Proceedings of the National Academy of Sciences, vol. 22 (1936), pp. 303–309.