SHORTER NOTICES

For the analytic discussion of the space P, rectangular Cartesian coordinates x, y, z are used (E is z=0) and homogeneous coordinates p_0, p_1, p_2, p_3 , where $x=p_2/p_1$, $y=p_3/p_1, z=p_0/p_1$. The exclusion of the line at infinity in E from P permits the normalizing condition $p_0^2+p_1^2=1$, so that we may write $p_0=\cos \lambda, p_1=\sin \lambda$; then λ, p_2, p_3 define a point in P or, equivalently, a displacement in E.

An algebra of displacements is obtained by means of biquaternions of the form $p = p_0 e_0 + p_1 e_1 + \epsilon (p_2 e_2 + p_3 e_3)$, where p_0, p_1, p_2, p_3 are real or complex numbers (the homogeneous coordinates as above) and $e_0, e_1, e_2, e_3, \epsilon$ satisfy $e_0 = 1, e_1^2 = e_2^2 = e_3^2 = -1$, $e_2 e_3 = -e_3 e_2 = e_1, \dots, \epsilon e_1 = e_1 \epsilon_1, \dots, \epsilon^2 = 0$. It is evident that the product of two biquaternions of the above form is a biquaternion of the same form. The conjugate \overline{p} of p is obtained by changing the signs of the coefficients of e_1, e_2, e_3 .

Now if a displacement p is applied to a rigid body, a point initially at l is displaced to r, where $r = \overline{p}lp$. If this displacement is followed by a second displacement q, then the final position of the point is t, where $t = \overline{p}^* lp^*$, in which $p^* = pq$; in fact, the resultant of two displacements is their product.

The "geometry" of the space P consists of those properties of figures in P which remain invariant under certain transformations. These transformations are as follows. Let us apply in order a displacement b_l , a displacement p, and a displacement b_r . The resultant is a displacement $p^* = b_l p b_r$. This is the required transformation of Pinto itself: these transformations form a group G_6 , since each of b_l , b_r depends on three parameters. These transformations leave invariant the points (0, 0, i, 1), (0, 0, 1, i)and the planes $x_0 \pm ix_1 = 0$, where x_0, x_1, x_2, x_3 are homogeneous coordinates. The resultant geometry the author calls *quasielliptic*, on account of a limiting connection with elliptic geometry.

The preceding remarks give some idea of the first half of the book (Algebraischer Teil). The second half of the book deals with the differential geometry of the quasielliptic space *P*. A curve is given by writing the homogeneous coordinates p_0 , p_1 , p_2 , p_3 as functions of a parameter λ . The privileged basic parameter is that λ which appears in connection with the normalizing condition $p_0^2 + p_1^2 = 1$, so that the equations of the curve are $p_0 = \cos \lambda$, $p_1 = \sin \lambda$, $p_2 = p_2(\lambda)$, $p_3 = p_3(\lambda)$. Quasicurvature and quasitorsion are defined. Surfaces are also considered and curves on surfaces.

The book is based on lectures delivered in 1938 at the University of Hamburg. One may smile at the encouraging prefatorial remark: "An Vorkenntnissen ist nur ein wenig analytische Geometrie und Infinitesimalrechnung erwünscht." For the book is not easy reading, seeming to lack purpose, so that the reader may well ask himself from time to time: "What is the goal of all this?" However, there is much meat in the little volume, as we might expect from the reputation of its author in geometry.

In addition to the regular exposition, there are a number of sections headed "Aufgaben und Lehrsätze."

J. L. Synge

Probability, Statistics, and Truth. By Richard von Mises. Translated by J. Neyman, D. Scholl, and E. Rabinowitsch. New York, Macmillan, 1939. 16+323 pp.

This translation follows closely the original, Wahrscheinlichkeit, Statistik und Wahrheit, 2d edition, 1936, which I reviewed in the Journal of the American Statistical Association (vol. 31 (1936), pp. 758–759). However, in the preface the author states, "I have added several paragraphs in the English edition (pp. 141–147). These deal with certain investigations of A. H. Copeland, E. Tornier, and A. Wald, which were published after the appearance of the second German edition."

The purpose of Mises is to present in language as non-technical as possible the

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