ON THE COMPUTATION OF THE SECOND DIFFERENCES OF THE Si(x), Ei(x), AND Ci(x) FUNCTIONS*

A. N. LOWAN

In the course of the computation of the functions†

$$(1)\quad \mathrm{Si}(x) = \int_0^x \frac{\sin\alpha}{\alpha}\,d\alpha = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)(2k+1)!},$$

$$(2)\quad \mathrm{Ci}(x) = \int_{-\infty}^x \frac{\cos\alpha}{\alpha}\,d\alpha = \gamma + \frac{1}{4}\log_e(x^4) + \sum_{k=1}^{\infty} (-1)^k \frac{x^{2k}}{2k\cdot(2k)!},$$

$$(3)\quad \mathrm{Ei}(x) = \int_{-\infty}^x \frac{e^\alpha}{\alpha}\,d\alpha = \gamma + \frac{1}{4}\log_e(x^4) + \sum_{k=1}^{\infty} \frac{x^k}{k\cdot k!},$$

it was felt advisable to precompute the second differences for the above functions. These second differences are of use in the Everett interpolation formula and may also be used as a check of the accuracy of the computed value. The object of this paper is to describe the method which was developed for the independent evaluation of the above second differences.

Let $\phi(x)$ stand for any of the three functions under consideration. Consider the expression

$$(4)\quad \begin{aligned} R(x) &= [\phi(x+h) + \phi(x-h) - 2\phi(x)] \\ &\quad - \frac{h}{2}[\phi'(x+h) - \phi'(x-h)], \end{aligned}$$

where the first expression in brackets is the second difference to be evaluated.

Substituting for $\phi(x+h)$, $\phi(x-h)$, $\phi'(x+h)$, and $\phi'(x-h)$ their Taylor expansions, we get

$$(5)\quad R(x) = \frac{-h^4}{12}\phi^{(4)}(x) + \sum_{k=3}^{\infty}\left[\frac{2}{(2k)!} - \frac{1}{(2k-1)!}\right]\cdot h^{2k}\phi^{(2k)}(x),$$

whence

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