

for the parameters, their weights and errors, the adjustment of the observations, and the confidence belts associated with the curve.

Lastly there are interesting exercises and notes on the formation of the normal equations for various functions to best fit certain measurements such as straight line, parabola, exponential, exponential with a lineal component, the generalized hyperbola together with three examples in curve fitting completely worked out, viz.:

Example 1, fitting an isotherm  $y = a + bx + cx^2 + dx^4$  with parameters subject to the condition  $x = 1$  when  $y = 1$ .

Example 2, the polynomial  $y = a + bx + cx^2$  with both  $x$  and  $y$  subject to error.

Example 3, an example useful in forestry, fitting  $x = ay^bz^c$  where  $x$  = volume of a tree,  $y$  = merchantable height of tree, and  $z$  = diameter at breast height, the data having been secured from 66 trees.

This treatise is to be commended for its completeness and for maintaining the single purpose of all work in least squares, the minimizing of  $\sum \overline{\text{res}}^2$ . Although the whole scheme is non-rigorous except for linear functions, the error introduced by ignoring the higher powers of the corrections to the assumed values for the parameters or other unknowns does not seriously affect the result. It seems to be the basis for a very good graduate course and shows clearly how the use of Lagrange multipliers may sometimes clarify an otherwise hopeless analytical problem.

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*Les Definitions Modernes de la Dimension.* By Georges Bouligand. Paris, Hermann, 1935. 44 pp.

This volume opens with a brief discussion of linear or affine geometry based on postulating the existence of an abstract system of elements called vectors which combine amongst themselves and with the real numbers to give us new elements of the same class. The dimension is determined by the number of linearly independent vectors. There is a brief discussion of spaces which in this sense would be of infinitely many dimensions.

Then denying the right of geometry to limit itself merely to spaces of this category, the author discusses three types of definition for dimension:

(1) The definitions which follow the direction of Fréchet who is interested in the spaces introduced by the needs of General Analysis. Fréchet develops a theory of dimension types or a topological frame based on the notion of homeomorphism between sets. His theory has as its axiomatic substructure three simple axioms concerning the closure of point sets.

(2) Those definitions where the fundamental notion is that of *separation* of sets by various subsets. These definitions emanate from the influence of Poincaré who started from the seeming contradiction of the existence of a (1-1) correspondence between the points of a line to which we should wish to assign the dimension one and the points of a plane which should be of dimension two. This line of development runs from Poincaré through Brouwer to Menger and Urysohn. The whole theory is based upon the three axioms fundamental in the Fréchet theory plus two additional axioms, that of normality and the second countability axiom, that is, a space in which distance can be defined. The author discusses the theorem of Lebesgue concerning intersection of coverings which can be used to give a definition of dimension independent of recurrence and which is usually used to show that the Menger-Urysohn definition gives the dimension  $n$  to sets to which we would intuitively assign the same dimension. A brief discussion is also given of the combinatorial point of view with respect to dimension which has been so ably developed by the powerful methods of Alexandroff.