SHORTER NOTICES

Some Notes on Least Squares. By W. E. Deming. Washington, D.C., The Graduate School, Department of Agriculture, 1938. 181 pp.

The author is conservative in calling this work *Some Notes on Least Squares.* It may not be what one would call a finished product judged by the standards of textbook requirements, but a reading will show it to be much more than the skeleton outline of a series of lectures on least squares.

At the outset we find what he calls *fragments in the history of least squares* which presents a fair picture of its development from Gauss' time, and with the copious references throughout the text one gets a thumbnail bibliography of the subject brought up to the present.

The treatment of the subject is logical and consistent throughout. It may be likened to what students of harmony call "variations on a theme"; and the author's theme is *The Minimizing of the Sum of the Squared Residuals* as enunciated by Gauss in his "theoris motus" (1809). It is interesting to note that in all the problems in least squares considered he begins the attack with this principle, adapting it to the special case by simple mathematical stages. In fact, he almost apologizes for the necessary use of the first derivative!

Believing that, in the present day, least squares cannot be disassociated from statistical researches that have been made since Gauss, the author has woven some of it into the material comprising these notes.

It is regrettable that this method of treatment does not suggest to the reader that, after all, the whole scheme is based on probability; that no mention is made of the probable error (as a modification of mean square error) still used by a considerable number of research and technical men in connection with the normal curve.

After a few simple applications in finding the best value from several direct measures, the author takes up the General Problem in Least Squares for which he gives a complete solution, whether it is a problem in curve fitting with adjustable parameters or one involving geometrical conditions or a combination of both cases, with all measured quantities subject to error.

The solution is made possible, of course, by approximation, since approximate values of the unknowns are necessary and the application of the principle of Gauss leads up to the normal equations involving corrections to the approximations as unknowns. There is nothing new in this idea, but most problems result in equations which are either solved with great difficulty or else defy analysis so that a clarifying device must be used if definite results are hoped for.

Commenting on this phase the author states—"All problems in least squares theoretically can be solved without the use of Lagrange multipliers. Occasionally it may be easier to dispense with them, but the truth is that most problems become hopelessly involved. The elegance and simplicity that they lend to all problems seem to me sufficient to displace all other possible methods in the design of a routine procedure. If Kummell in 1879 had introduced Lagrange multipliers, he would surely have accomplished the general solution that he was looking for."

This solution may be easily adjusted to any set of conditions simple or complex: for instance, if some of the measured quantities are free from error, this is taken care of by the simple expedient of making their respective weights infinite.

In the section on curve fitting the plan is outlined for obtaining the best values