SPACES OF UNCOUNTABLY MANY DIMENSIONS*

C. W. VICKERY

Riemann in his *Habilitations Schrift* of 1854 suggested the notion of n-dimensional space (where n is a natural number) as an extension of the notion of three-dimensional euclidean space. Hilbert extended the notion still further by defining a space of a countably infinite number of dimensions. Fréchet† in 1908 defined two other spaces of countably many dimensions, which he called D_ω and E_ω. Tychonoff‡ in 1930 defined a series of spaces of an unlimited number of dimensions and established several of their properties. The present paper undertakes, by generalizing the notions of spaces D_ω and E_ω, to define spaces D^α and E^α, respectively, for each cardinal number $\aleph_\alpha$ representing the number of dimensions. It is shown that every metric space is homeomorphic with a subset of some space D^α. Certain properties of Tychonoff's spaces, here called spaces T^α, are also presented.

1. **Spaces** D^α. For each initial ordinal number ω_α the set of all points of space D^α is the set of all type ω_α sequences $[x_i]_i^{\omega_\alpha}$ of real numbers x_i, such that $0<x_i<1$. Suppose that $[P_i]_i^{\omega_0}$ is a type ω_0 sequence of points of space D^α such that for each i, $P_i=[x_{i,j}]_j^{\omega_\alpha}$; and suppose that $P=[y_j]_j^{\omega_\alpha}$ is a point of space D^α. The sequence $[P_i]_i^{\omega_0}$ is said to *converge* to the point P if and only if it is true that if ϵ is a positive number, there exists a positive integer N_ϵ such that if $i>N_\epsilon$, then $|y_j-x_{i,j}|<\epsilon$ for every value of $j<\omega_\alpha$. The point P is said to be the *sequential limit point* of $[P_i]_i^{\omega_0}$. A point P is said to be a *limit point* of a point set M, provided there exists a sequence of distinct points of M which converges to P. Thus space D^0 is equivalent to space D_ω of Fréchet. For each two distinct points $A=[x_i]_i^{\omega_\alpha}$ and $B=[z_i]_i^{\omega_\alpha}$ of D^α such that for each i, $x_i\neq z_i$, let $D^\alpha_{(A,B)}$, also referred to as *segment* AB, denote the set of all points $P=[y_i]_i^{\omega_\alpha}$ such that $x_i<y_i<z_i$ or $z_i<y_i<x_i$. If there exist constants h and k, $(h<k)$, such that for each i, $x_i=h$ and $y_i=k$, the notation $D^\alpha_{(h,k)}$ is used. It is evident that $D^\alpha_{(h,k)}$ is homeomorphic with D^α for every value of α, h, and k; but there exist points A and B such that $D^\alpha_{(A,B)}$ is not homeomorphic with D^α.

* Presented to the Society, September 12, 1935, and October 26, 1935 (under the title *Concerning spaces of uncountably many dimensions*).

† M. Fréchet, *Essai de géométrie analytique à une infinité de coordinées*, Nouvelles Annales de Mathématique, (4), vol. 8 (1908), pp. 97–116. Also *Les Espaces Abstraits*, Paris, 1928, pp. 81–84, 97–99.

‡ Mathematische Annalen, vol. 102 (1930), pp. 544–561.