ON THE SINGULAR POINT LOCUS IN THE THEORY OF FIELDS OF PARALLEL VECTORS

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The surface of an ordinary (circular) half cone is frequently employed as an illustration of a surface on which parallel displacement of a vector around a closed circuit may produce a change in the vector. It may appear at first sight that the reason for this is related to the fact that the circuit in question encircles the vertex of the cone and the vertex is in some sense a singular or exceptional point. But this cannot be correct since we can remove the vertex from the surface under consideration without effecting the parallel displacement of the vector. If it is then argued that the resulting surface is not simply connected and that the above phenomenon depends on the fact that the circuit along which the vector is displaced cannot be shrunk to a point, we can offset this by smoothing out the surface in the neighborhood of the vertex so that it becomes a continuous, differentiable, and simply connected surface (which is evidently possible). Clearly then, such considerations do not provide us with the inner reason as to why the parallel displacement of a vector around certain closed circuits on the surface yields the original vector while for other circuits a different vector is obtained as result of the parallel displacement.

A satisfactory answer to the above question is contained in a general theory of the parallel displacement of vectors in an affinely connected space by Mayer and Thomas, *Fields of parallel vectors in non-analytic manifolds in the large*, Compositio Mathematica, vol. 5 (1938), pp. 193–207. It can be shown that an affinely connected space of class C^r, where $r \geqq n+1$ and n is the dimensionality of the space, breaks up *in virtue of its intrinsic nature* into a finite or infinite number of open point sets K, called components, with each of which there is associated a definitely determined integer m having a value from zero to n inclusive. Denote by K_m any component K for which m is the associated integer. Then any open, connected, and simply connected point set $O \subset K_m$ admits exactly m independent fields of parallel vectors. The definition of the components K is as follows: Consider the set of equations

$$(E_0)\xi^{\mu} B^{\alpha}_{\mu\beta\gamma} = 0; \; (E_1)\xi^{\mu} B^{\alpha}_{\mu\beta\gamma,\delta_1} = 0; \cdots ; \; (E_n)\xi^{\mu} B^{\alpha}_{\mu\beta\gamma,\delta_1,\cdots,\delta_n} = 0$$

as equations for the determination of the n quantities ξ^{μ}, the co-