A TENSOR ANALYSIS FOR A V_k IN A PROJECTIVE SPACE S_n*

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1. **Introduction.** In this paper we shall show how an intrinsic tensor analysis may be developed for a curved space or variety V_k of k dimensions immersed in a projective space S_n of $n>k>1$ dimensions. For $n \neq k-1$ there apparently exists no covariant quadratic differential form; so Fubini's method of studying such a variety fails. Or, as Lane suggests,† Fubini's method fails either due to the lack of a quadratic differential form, or to the lack of an absolute calculus for an n-ary p-adic form except when $p=2$.

However, it is well known that an absolute calculus can be developed without the use of a quadratic form by making use of certain generalized Christoffel symbols.‡

These three indexed symbols enable one to introduce into the geometrical theory of a variety the geometry of paths, affine and "projective" connections. In that manner certain tensors and vectors arising in those theories can be expressed in terms of tensors and vectors arising in the study of the variety from the point of view of classical projective geometry. In particular, the Weyl projective curvature tensor is expressible in terms of tensors arising in the classical geometric theory of a variety V_k.

Finally, we show that a generalized Riemann space of k dimensions with a fundamental symmetric connection characterizing the space may be considered as being immersed in a projective space of $n=k(k+3)/2$ dimensions. This theorem is an evident generalization of the fact that a Riemann space may always be considered as immersed in an euclidean space of sufficiently high dimension.

2. **The fundamental differential equations.** Let the homogeneous projective coordinates x^i, $(i=1, 2, \cdots, n+1)$, of a point P in S_n be given as analytic functions of exactly k parameters $u^1, u^2, \cdots, u^k$:

$$x^i = x^i(u^1, u^2, \cdots, u^k). \tag{1}$$

The totality of such points P we shall call *a variety* V_k.

The functions x and $\partial x/\partial u^\rho$ may be interpreted as the homogeneous projective coordinates of $k+1$ points. These points determine a cer-

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† Lane [1, p. 292].

‡ See, for example, Eisenhart [2].